The Dynamical Hypothesis in Cognitive Science

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Abstract

The dynamical hypothesis is the claim that cognitive agents are dynamical systems. It stands opposed to the dominant computational hypothesis, the claim that cognitive agents are digital computers. This target article articulates the dynamical hypothesis and defends it as an open empirical alternative to the computational hypothesis. Carrying out these objectives requires extensive clarification of the conceptual terrain, with particular focus on the relation of dynamical systems to computers.

Key words

cognition, systems, dynamical systems, computers, computational systems, computability, modeling, time.

Long Abstract

The heart of the dominant computational approach in cognitive science is the hypothesis that cognitive agents are digital computers; the heart of the alternative dynamical approach is the hypothesis that cognitive agents are dynamical systems. This target article attempts to articulate the dynamical hypothesis and to defend it as an empirical alternative to the computational hypothesis. Digital computers and dynamical systems are specific kinds of systems. The dynamical hypothesis has two major components: the nature hypothesis (cognitive agents *are* dynamical systems) and the knowledge hypothesis (cognitive agents can be understood dynamically). A wide range of objections to this hypothesis can be rebutted. The conclusion is that cognitive systems may well be dynamical systems, and only sustained empirical research in cognitive science will determine the extent to which that is true.

1. Introduction

Some five decades after *Principia Mathematica*, David Hume dreamt of a scientific psychology in which mathematical laws would govern the mental realm, just as Newton's laws governed the material realm (Hume, 1978). The universal force of gravitation, whereby bodies attract in proportion to their masses, would be replaced by a universal force of association, whereby ideas attract in proportion to their similarity. The dynamics of matter would be paralleled by a dynamics of mind.

The Humean dream was not the first vision of mind inspired by the emergence of modern science. The new physics had uncovered mathematical laws of great simplicity and elegance, but laborious calculation was required to derive the messy details of actual behaviors. Thomas Hobbes took this calculating activity itself as his model of the mechanisms of mental operation. Perhaps thought is symbolic computation, the rule-governed manipulation of symbols inside the head (Hobbes, 1651/1962).

Seventeenth-century speculation became twentieth-century science. Hobbes's idea evolved into the *computational hypothesis* (CH), that cognitive agents are basically digital computers. Perhaps the most famous rendition is Newell and Simon's (1976) doctrine that "A physical symbol system has the necessary and sufficient means for general intelligent action." They proposed this hypothesis as a "law of qualitative structure," comparable to the cell doctrine in biology or plate tectonics in geology. It expresses the central insight of the research paradigm which has dominated cognitive science for some forty years.

In recent years, however, the Humean alternative has been gaining momentum. One of the

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most notable developments has been the rise of connectionism, which models cognition as the behavior of dynamical systems (Smolensky, 1988), and often understands those models from a dynamical perspective. Equally significant is the emergence of cognitive neuroscience, and within it, the increasing prevalence of dynamical theorising. Dynamics forms the general framework for growing amounts of work in psychophysics, perception, motor control, developmental psychology, cognitive psychology, situated robotics and autonomous agents research, artificial intelligence, and social psychology. It is central to a number of general approaches, such as ecological psychology, synergetics, and morphodynamics.[1]

The *dynamical hypothesis* (DH) is the unifying essence of dynamical approaches to cognition. It is encapsulated in the simple slogan, *cognitive agents are dynamical systems*. The aims of this target article are (1) to *articulate* the hypothesis—i.e., to explain what the slogan means—and (2) to *defend* it as an open empirical hypothesis standing as a substantive alternative to the CH. The DH contends for the status of the “law of qualitative structure” concerning the nature of cognition.

One goal in undertaking this philosophical work is to clarify the conceptual terrain. Another is to help clear rhetorical space for dynamicists in cognitive science to get on with the hard work of developing detailed accounts of specific cognitive phenomena. The most important goal, however, is to gain insight into the nature of people—for people are, among other things, cognitive agents.

This paper ploughs an interdisciplinary field. Boulders of ambiguity, vagueness and confusion must be cleared away. Much effort is devoted simply to establishing a single coherent and reasonably precise framework for discussion. This framework involves commitments at terminological, conceptual and even metaphysical levels. Its development requires many choices and stipulations, often somewhat arbitrary in nature. Occasional conflicts with existing intuitions are unavoidable. Still, *some* such regimentation is essential, for otherwise debating the DH is just a futile exercise in miscommunication. Table 2 in the appendix summarizes the framework by listing key terms and their meanings as deployed here.

2. Some Examples

A first task is to sketch some representative examples of dynamical cognitive science, to serve as a backdrop for the following discussion. Space limits dictate brevity; readers are encouraged to visit the original sources for proper treatment.

Consider how we come to make choices between actions with various possible outcomes. If we were digital computers, we would symbolically represent to ourselves the various options and their outcomes, together with our estimates of the likelihood of those outcomes and their value to us. Reaching a decision would then be a matter of calculating the most promising option. An alternative Humean account has been proposed by psychologists Jerome Busemeyer and Jim Townsend (Busemeyer & Townsend, 1993). In their “Decision Field Theory” (DFT) model, relevant aspects of the decision situation are represented not by

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[1] Examples: cognitive neuroscience: (Amit, 1989; Babloyantz & Lourenco, 1994; Cohen, 1992; Guckenheimer, Gueron, & Harris-Warrick, 1993; Mpitsos, forthcoming; Skarda & Freeman, 1987); psychophysics: (Gregson, 1995); perception: (Bingham, Rosenblum, & Schmidt, in press; Grossberg & Rudd, 1992; McClelland & Rumelhart, 1981; Port, Cummins, & McAuley, 1995); motor control: (Bullock & Grossberg, 1988; Saltzman, 1995; Turvey, 1990); developmental psychology: (Smith & Thelen, 1993; Thelen & Smith, 1993); cognitive psychology: (Busemeyer & Townsend, 1993; Grossberg & Gutowski, 1987; Grossberg & Stone, 1986; Leven & Levine, 1996; Tabor, Juliano, & Tanenhaus, 1996); situated robotics and autonomous agents research: (Beer, 1995b; Cliff, Harvey, & Husbands, 1993; Smithers, 1994); artificial intelligence (Jaeger, 1996; Pollack, 1991); social psychology (Kaplowitz & Fink, 1992; Vallacher & Nowak, 1993); ecological psychology (Kugler, Kelso, & Turvey, 1980; Kugler, Kelso, & Turvey, 1982; Turvey & Carello, 1995); synergetics (Haken & Stadler, 1990); morphodynamics: (Petitot, 1985b; Thom, 1983; Wildgen, 1982). (Port & van Gelder, 1995) is a representative sampling of the dynamical approach. Note that works cited here are intended as examples and pointers, rather than any kind of exhaustive or definitive listing.

symbols but by means of continuous quantities. Decision-making is the interdependent evolution of these quantities over time as governed by mathematical equations (as opposed to algorithms). Decisions are made when certain thresholds are passed. The scientific question then is: Which kind of model best accounts for the actual psychological data on human decision-making? Busemeyer & Townsend claim that their model predicts actual decisions better than any “static-deterministic” model, as well as describing temporal properties of decision processes beyond the scope of traditional models.

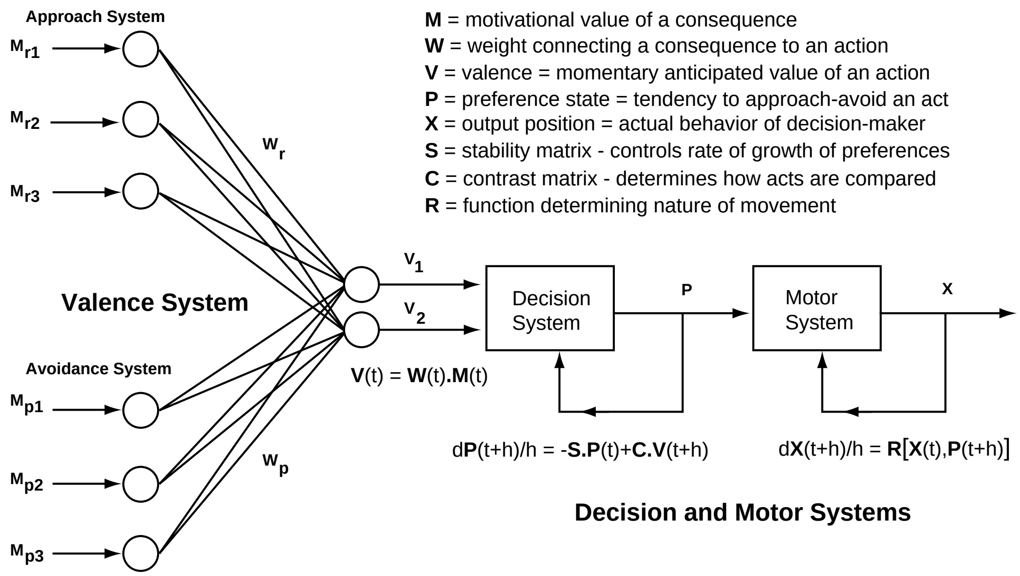

Figure 1. Outline of the “Decision Field Theory” dynamical model of decision-making processes (Busemeyer & Townsend, 1993). The decision-making process begins (far left) with a set of possible gains and losses (*M*), filtered by attentional weights (*W*) to form the valence (momentary anticipated value, *V*) of an action. The decision system temporally integrates the valences to produce a preference state for each action (*P*). The preferences drive a motor system producing an observed action. (Figure and legend adapted from (Busemeyer & Townsend, 1995).)

For an example of a very different kind, consider how we manage to move our limbs. A Hobbesian would maintain that we calculate how and when to contract muscles, much as a digital computer lands a 747 by calculating engine thrust, flap angle, etc.. A dynamical alternative has been under development by Scott Kelso and coworkers. His classic example is coordinating the wagging of your index fingers. Performance on this task has some remarkable properties. At low wagging speeds there are two comfortable coordination patterns, inphase and antiphase (bistability). As speed is gradually increased, anti-phase patterns start to lose their stability; eventually a point comes where only inphase patterns are stable (bifurcation). As speed decreases, antiphase patterns become possible again, but not until somewhat below the original collapse point (hysteresis). Kelso found that these and other properties can be described and predicted in detail by assuming that a single, continuous, high level, “collective” variable—relative phase—evolves in a way governed by a suitable form of a simple differential equation (Kelso, 1995, p. 55).[2] Variants of this “HKB”

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[2] The basic Haken-Kelso-Bunz equation is

$$\dot{\phi} = -a\sin\phi - 2b\sin 2\phi$$

Here ϕ is the single “collective” state variable of the system; in the finger coordination model, it corresponds to the oscillation phase of one finger relative to the other. The model have been applied in diverse cognitive domains.[3] The basic insight is that coordination is best thought of as explained not as masterminded by a digital computer sending symbolic instructions at just the right time, but as an emergent property of a nonlinear dynamical system self-organizing around instabilities.

These models purport to provide the best available empirical accounts of phenomena in their domains. Whether they succeed is an interesting question for specialists to address. What matters here is that they nicely illustrate the dynamical approach to cognition.

3. Systems, Digital Computers, and Dynamical Systems

A critical step in articulating the DH is stating, in a reasonably precise yet flexible way, just what dynamical systems are. A useful approach is to distinguish dynamical systems and digital computers as different kinds of *systems*.

3.1 Systems

Systems are here taken to be sets of interdependent variables.[4] A *variable* is simply some entity that can change, i.e., be in different states at different times. Variables are *interdependent* when the way any one changes depends on others, and change in others depends on it. The *state* of the system is simply the state or value of all its variables at a time; the *behavior* of the system consists of transitions between states.

For example, the solar system of classical mechanics is the set of positions and momentums of the sun and planets; these are the quantities whose behaviors are described by Newton's laws. Note that the variables of the solar system in this sense are *properties* of the sun and planets. We must therefore distinguish objects (parts of the world such as the sun and planets, Macintoshes, and cognitive agents) from the systems they *instantiate*. Any given object will usually instantiate a great many systems of different kinds.

Concrete systems are those, like the solar system, whose variables are actual features of the real world changing in real time in accordance with natural laws. *Abstract* systems are just sets of abstract variables governed by mathematical rules. Concrete systems can *realize* abstract systems. For example, two HP61 calculators realize exactly the same abstract computational system.

Concrete systems are slices of the causal organization of nature. Causal organization comes in many kinds and at many levels. Distinct systems can be intimately related. Compare the classical solar system with the system made up of all the positions and momentums of all their constituent subatomic particles. The (macro)variables of the former are built up out of the (micro)variables of the latter. The relationship between these systems is neither identity[5]

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equation specifies how relative phase changes as a function of its current value. *a* and *b* are parameters of this system; their ratio corresponds to the rate of wagging of the fingers. The equation is such that gradual changes in *a* and *b* can yield just the kind of qualitative changes in relative phase found in the behavior of real subjects.

This simple "frictionless" equation is altered in various ways to generate models with better fit to experimental data. For example, fluctuations and symmetry-breaking considerations are accomodated by adding terms for noise and for differences in frequency between limbs and metronome (Kelso, DelColle, & Schöner, 1990).

3 These include aspects of motor skill learning (Schöner, Zanone, & Kelso, 1992), interpersonal coordination (Schmidt & Turvey, 1994), speech perception (Tuller, Case, Mingzhou, & Kelso, 1994), and visual perception (Hock, Kelso, & Schöner, 1993). See (Kelso, 1995) for an overview.

4 This definition accords with ordinary usage (e.g., Websters Dictionary: "a regularly interacting or interdependent group of items forming a unified whole") and systems theory (e.g., "a set of elements standing in interrelations," (von Bertalanffy, 1973), p.55). The stance on the metaphysical status of sets adopted here is the "set-theoretic realism" elaborated in (Maddy, 1990). In this account, sets of physical entities are themselves physical entities, as much part of the ordinary world as planets, people and PCs.

5 In set theory, set identity is a matter of having exactly the same members. A set of sets is

nor instantiation. In what follows, a lower-level system will be said to *implement* a higher-level system when the variables of the latter are somehow constructed out of variables of the former. Note that implementation licenses us to identify the behavior of the one system with the behavior of the other, despite failure of strict identity between the systems themselves.

Often, change in a system depends on factors outside the system itself (e.g., the force of gravity), referred to here as *parameters*. Sometimes, changes in a parameter depend in turn on the system itself. For example, the position of the moon both depends upon, and affects, the position of the planets. This kind of reciprocal, direct dependence is known as *coupling*. System variables and coupled parameters can be regarded as forming a larger system. This illustrates the semi-arbitrariness of systems. It is always up to us to nominate a set of concrete variables as the system we will study. Reality determines whether that set is in fact a system, and how it behaves.

All systems in the current sense change in time. In general, *time* is just some intrinsically ordered set, or *order*[6], serving to provide orderings over other things. The real time of concrete systems is the set of instants at which things can actually happen, ordered by temporal priority (before/after). Concrete events are paired with instants or periods of time, and hence stand in temporal relations with each other. Abstract systems are not situated in real time at all, and so must take some other set as their time set; usually, it is the positive integers or the real numbers. The mathematical rule imposes orderings over states of the system by pairing them with members of this set.

3.2 Digital Computers

The CH has benefitted from considerable philosophical scrutiny. One result is a remarkable level of consensus over its basic commitments[7]. In particular, it is widely agreed to maintain that cognitive agents are *digital computers*. But what is a digital computer, as a kind of system?

A *computer* is simply anything that computes in some way or other. *Computing* is an informal notion; the basic idea is that of a process systematically transforming “questions” into “answers”— inputs into outputs, start states into final states, etc.. The *function* computed by that process is the set of question/answer pairs themselves, or the set of pairs of entities they represent. In this general sense pretty much anything can be construed as a computer. Computation only gets interesting when significant constraints are placed on the kinds of processes involved. In classical computation theory, the standard approach has been to require that processes be *effective*, i.e., produce their results by means of a finite number of basic operations specified by an algorithm (a finite recipe, or set of instructions specifying basic operations).

Digital computers, in the sense that matters for cognitive science, are systems which carry out effective computation over representations. That is, they are systems whose behaviors are algorithmically specified finite sequences of basic operations constituting manipulations of representations. This characterization can be broken down into four fundamental requirements on a system to count as a digital computer:

(1) *Digital variables and states*. First, for each variable there must be some set of *discrete* values which the variable instantiates *digitally* for the purposes of system behavior. In the

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not identical with the set of the elements of those sets. Thus, strictly speaking, a set of pairs of socks is not identical with the set of socks belonging to those pairs. Of course, there is still an obvious and important sense in which these sets are the same. In this paper, this sense is captured by the notion of *implementation*.

6 A non-empty set X is an *order*, or is *ordered*, if there is a relation $<$ over its elements with the property that for each $x,y \in X$, either $x < y$, or $y < x$, or $x = y$.

7 For expressions of this consensus see, for example, (Clark, 1989; Copeland, 1993; Dreyfus, 1992; Fodor, 1975; Fodor & Pylyshyn, 1988; Newell, 1980; Newell & Simon, 1976; Pylyshyn, 1984). The version of this consensus now most widely accepted as definitive is probably that laid out in (Haugeland, 1985). The account of digital computers here is essentially just Haugeland’s definition of computers as interpreted automatic formal systems as massaged into the present framework.

concrete case, this means that the variable must instantiate those variables positively and reliably.[8] When all variables in a system are digital, the system's *states* are also digital. The basic operations required by effective computation correspond to digital state transitions.

(2) *Time as discrete order*. The time set must be a discrete order whose elements are the times at which the system digitally occupies its states. In abstract systems, this is usually the positive integers. In concrete systems, it is the set of periods of real time at which the machine digitally instantiates its states, as rendered discrete by the flux of transition between states. These are indexed by the positive integers (t_1, t_2, etc.).

(3) *Algorithm*. Effective computation requires basic operations to be specified by an algorithm, i.e., a finite recipe specifying state transitions solely on the basis of digital properties of states. For example, the infinite range of behaviors of a Turing Machine is governed by its machine table, a finite set of instructions expressed only in terms of the digital values of tape squares, head position, and head state. In concrete systems, this rule must capture one level of causal organization. That is, the transitions described by the rule must happen the way they do *because* the states bear the digital properties in terms of which the rule is expressed.

(4) *Interpretation.* The system's states and behaviors must yield to systematic interpretation. That is, there must be some domain, and correspondences between the system and the domain, such that (a) the correspondences are *systematic* with respect to those digital aspects of the system in terms of which the rule governs system behavior, and (b) the system's states and behaviors *make sense* in the light of those correspondences.[9]

The distinction made above (S.3.1) between the solar system of classical mechanics on one hand and the sun and planets on the other is mirrored by a distinction between digital computers and the ordinary notion of computers as what you take out of the box and plug into the wall. The digital computer *system* is the object of theoretical interest. The hunk of silicon, plastic, glass, metal, etc., *instantiates* some digital computer (system), and of course many other systems as well.

3.3 Dynamical Systems

By comparison with the CH, the DH has been starved of attention.[10] Partly as a result, there is no established consensus over what dynamical systems are for the purposes of the hypothesis. Unfortunately, there is also a wide range of definitions in mathematics and science more generally (Table 1). These range from older, narrow definitions in terms of particles governed by forces to more recent broad definitions which subsume all systems in the current sense. There is no single official definition waiting to be lifted off the shelf. Nevertheless, cognitive scientists do have a good working grasp of the issue. In the vast majority of cases they agree whether a system counts as dynamical in the sense that matters for them. The challenge here is to articulate that intuitive understanding.

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[8] See (Haugeland, 1985), Chapter 2. In abstract systems, discreteness of values suffices for digitality.

[9] What is it to "make sense"? This is a difficult issue; see (Haugeland, 1985), Chapter 3, for discussion. Every digital system can be set up in systematic correspondence with some domain (such as integers and functions over them) but not all such systems have an interpretation in the current sense. The ones that do are those exhibiting a further kind of order that does or could seem patterned or reasonable *to us* (humans); thus, whether something is a digital computer is human-relative.

Note that having an interpretation in the current sense may not be enough to guarantee that the system has "meaning" in some stronger sense, (and hence, perhaps, "mind"). For discussion of these issues, see Harnad (1990) and Searle (1980).

[10] Recently, philosophers have begun to repair this neglect. See, for example, (Giunti, forthcoming; Horgan & Tienson, 1996; van Gelder, 1995; van Gelder & Port, 1995) for discussion more or less closely related to the current issues.

Table 1. *Some examples of common definitions of the term "dynamical system" from outside cognitive science, arranged roughly in order, from older narrower definitions to more recent wider ones.*

Guiding Idea	**Examples**
1. A system of bodies whose motions are governed by forces. Such systems form the domain of dynamics considered as a branch of classical mechanics.	"a collection of a large number of point particles." (Desloge, 1982) p.215 Webster's: "dynamics...a branch of mechanics that deals with forces and their relation primarily to the motion... of bodies of matter."
2. A physical system whose state variables include rates of change	"In the original meaning of the term a dynamical system is a mechanical system with a finite number of degrees of freedom. The state of such a system is usually characterized by its position...and the rate of change of this position, while a law of motion describes the rate of change of the state of the system." (1989) p.328
3. A system of first-order differential equations; equivalently, a vector field on a manifold	a dynamical system is "simply a smooth manifold *M*, together with a vector field *v* defined on *M*." (Casti, 1992) p.109
4. Mapping on a metric space	"A *dynamical system* is a transformation $f:\boldsymbol{Z}\rightarrow\boldsymbol{Z}$ on a metric space $(\boldsymbol{Z}, d)$." (Barnsley, 1988) p.134.
5. State-determination	"a dynamical system...is one whose state at any instant determines the state a short time into the future without any ambiguity." (Cohen & Stewart, 1994) p.188
6. Any mapping, equation, or rule.	A dynamical system may be defined as a deterministic mathematical prescription for evolving the state of a system forward in time." (Ott, 1993) p.6
7. Change in time	"A dynamical system is one which changes in time." (Hirsch, 1984) p.3 "The term *dynamic* refers to phenomena that produce time-changing patterns...the term is nearly synonymous with time-evolution or pattern of change." (Luenberger, 1979) p.1

An obvious feature distinguishing dynamical models in cognitive science from standard computational models is that their variables are *numerical*. One reason numbers are so useful in science is that they have *quantitative* properties. This suggests that dynamical systems in cognitive science might be defined as quantitative systems. Roughly, a system is quantitative when there are *distances* in state or time, such that these distances matter to behavior. This can be true in progressively deeper ways, giving rise to progressively more substantial senses in which a system can count as dynamical.

(1)*Quantitative in state.* First, there can be distances between any two overall states of the system, such that the behavior of the system depends on these distances. More precisely, a system is quantitative in state when there is a metric[11] over the state set such that behavior is systematically related to distances as measured by that metric. Such systems will be governed by a rule compactly specifying this distance-dependent change. For example, the difference equations in the DFT model describe how the system changes by telling us the distance between the values of variables at time t and their values at time $t+h$.

Standardly, the relevant quantitive properties of state sets are derived from quantitive properties of the variables. Quantitive variables can be either abstract or concrete. For example, the variable ϕ in the HKB model is an abstract mathematical magnitude whose values are real numbers. This variable corresponds (via measurement; see Krantz, Luce, Suppes, & Tversky, 1971) to a concrete quantity whose values are relative phases of oscillation of index fingers. The model works precisely because the quantitative properties of the concrete variable are reflected in the quantitative properties of the abstract counterpart.

(2) *Quantitative state/time interdependence.* A system is quantitative in *time* when time is a quantity, i.e., there is a metric over the time set, such that system behavior is systematically related to distances as measured by that metric. At least in cognitive science practice, systems that are quantitative in time are also quantitative in space, and these properties are interdependent. That is, the behavior of the system is such that *amounts* of change in state are systematically related to *amounts* of elapsed time. Such systems are governed by a rule specifying a quantitative relationship between change in state, elapsed time, and current state. In concrete systems, this rule captures causal organization; that is, the system changes as it does because system variables have the quantitative properties in terms of which the rule is expressed. When both state and time are quantitative, the system exhibits *rates* of change. Systems that are interdependently quantitative in state and time are governed by rules specifying the rate of change in terms of current state (e.g., first-order differential equations).

(3)*Rate dependence.* Third, some systems are such that their rates of change depend on current rates of change. In these systems, variables include both basic variables and the rates of change of those variables. The solar system is a classic example. Systems whose behavior is governed by rules most compactly expressed as sets of higher-order differential equations are quantitative in this sense.

In what follows, a system is taken to be dynamical to the extent that it is quantitative in one of the above senses.[12] At least four considerations support this approach. First, it reflects the actual practice of cognitive scientists in classifying systems as dynamical or not, or as more or less dynamical. Second, it sits comfortably with existing definitions. The levels of quantitative character roughly correspond to definitions 1-4 of Table 1. Third, it is cast in terms of deep, theoretically significant properties of systems. For example, a system that is quantitative in state is one whose states form a *space*, in a more than merely metaphorical sense; states are *positions* in that space, and behaviors are paths or trajectories. Thus quantitative systems support a geometric perspective on system behavior, one of the hallmarks of a dynamical orientation. Other fundamental features of dynamical systems, such as stability and attractors, also depend on distances. Fourth, the definition sets up a contrast between dynamical systems and digital computers (see Section 6). For these reasons, defining dynamical systems as quantitative systems facilitates articulation and defense of the DH.[13]

4. The Dynamical Hypothesis

What does it mean to say that cognitive agents are dynamical systems? First, note that the

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11 A metric over a set X is a function $d{:}X\times X \to \mathbf{R}$ that assigns to every pair of elements x and y a number $d(x,y)$ 0 such that $d(x,y)=0$ iff $x=y$, $d(x,y) = d(y,x)$, and $d(x,y)$ $d(x,z) + d(z,y)$.

12 This formulation is designed to accomodate some rather special cases of dynamical systems whose behavior is generally quantitative except at certain isolated points (Gregson, 1993; Zak, 1990).

13 The concept of *dynamical system* changes over time, in cognitive science as elsewhere. Future developments might prompt broadening of the current definition. For example, cognitive scientists *may* come to use as models systems whose state sets are not metric spaces, but do possess some other kind of interesting topological structure relevant to system behavior.

hypothesis has two major components. The *nature* hypothesis is a claim about the nature of cognitive agents themselves; it specifies what they *are* (i.e., dynamical systems). The *knowledge* hypothesis a claim about cognitive science: namely, that we can and should *understand* cognition dynamically. Obviously, these are closely related; the best evidence for the former would be the truth of the latter. Nevertheless, they make different claims, and are best elaborated separately.

First, some preliminary points. The proper domain of the DH is *natural* cognitive agents—i.e., evolved, biological agents such as people and other animals. It need take no stand on the possibility of artificial cognition in digital computers. Second, the DH is limited in its explanatory pretensions. It is concerned only with the causal organization of agents insofar as they exhibit cognitive performances. Other forms of explanation may also be deeply illuminating. For example, evolutionary explanations might best explain *why* an agent has a particular causal organization.

What is it to be *cognitive*? In the most traditional sense, cognitive processes are those involving *knowledge*; cognitive science would then be the study of knowledge-based processes. However, as cognitive science has matured it has diversified. Knowledge is now only one indicator of cognitive status; others include intelligence, adaptability, and coordination with respect to remote states of affairs. The concept now resists capture in terms of any concise set of strict conditions. This paper simply takes an intuitive grasp of the issue for granted. Crudely put, the question here is not what makes something cognitive, but how cognitive agents *work*.

4.1 The Nature Hypothesis.

The nature hypothesis tells us what cognitive agents are by specifying the relation they bear to dynamical systems. It is common to interpret the hypothesis as asserting that cognitive agents are literally identical with some particular low-level system made up of a large number of internal, low-level quantities such as neural firing rates. However, this needs correction in almost every respect.

First, the relationship at the heart of the nature hypothesis is not identity but *instantiation*. Cognitive agents are not themselves systems (sets of variables) but rather objects whose properties, etc., can form systems. Cognitive agents instantiate numerous systems at any given time. According to the nature hypothesis, the systems responsible for cognitive performances are dynamical.

Second, cognitive agents "are," in this sense, not some particular dynamical system, but as many systems as are needed to produce all the different kinds of cognitive performances exhibited by the agent. Consider the DFT and HKB models from Section 2. These models invoke quite different sets of variables. One suggests that cognitive agents make decisions by virtue of change in valences, preferences, etc.; the other, that cognitive agents coordinate finger movements by virtue of change in relative phase. These models are not in competition. Both might be complete accounts of phenomena in their respective domains, implying that cognitive agents are many dynamical systems at once.

Another noteworthy fact about these models is that the variables they posit are not low-level (e.g., neural firing rates), but rather macroscopic quantities at roughly the level of the cognitive performance itself. The lesson here is that the nature hypothesis is concerned in the first instance not with low-level systems but with how agents are causally organized at the highest level relevant to an explanation of cognitive performances, whatever that may be.

Finally, notice that the DFT model includes not only "internal" variables such as preferences and valences, but also the "position" of the agent. More generally, the dynamical system responsible for a given kind of cognitive performance might include variables not literally contained within the agent itself, on any ordinary conception of its boundaries. For example, ecological psychologists understand visually guided locomotion as change in a dynamical system which includes aspects of both the organism and the environment (e.g., the optic flow; Warren, 1995).

4.2 The Knowledge Hypothesis.

It is one thing for cognitive agents to *be* dynamical systems, but it is quite another for us to *understand* them as such. The knowledge hypothesis is the bold claim that cognitive science *can* and *should* take dynamical form. What does this involve?

4.2.1 Dynamical Models

Given something we wish to understand—an explanatory *target*—a model is some *other* thing, relevantly similar but somehow more amenable to investigation. Understanding of the model transfers to the target across the bridge of similarity. Note that often the full complexity and detail of the target will defy human comprehension. In such cases, a model provides scientific insight precisely because it is a simplification.

One of the most common strategies in science is the use of abstract dynamical systems as models. The dynamical approach to cognition follows in this tradition. The performance of interest is taken to be interdependent change in some concrete dynamical system instantiated by the agent. The scientist furnishes an abstract dynamical system to serve as a model by specifying abstract variables and governing equations. Simple models can be fully understood by means of purely mathematical techniques. More commonly, however, scientists enlist the aid of digital computers to *simulate* the model (i.e., compute approximate descriptions of its behavior). The simulation results are compared against experimental data from the target. To the extent that the correspondence is close, the target system is taken to be similar in structure to the abstract dynamical model. Note that the digital computer, since it is not itself a dynamical system (for explanation of this claim, see Section 6.2), is not similar in the relevant sense to the target system, and so is not a model of it. We do not attempt to understand the target *by* understanding the digital computer; rather, we use the computer as a tool in our attempt to understand the target *by* understanding the abstract model.

The distinctive flavor of Humean dynamical modeling is enhanced by juxtaposition with its Hobbesian counterpart (Figure 2). In both cases, there is a target system, an abstract model, and a digital computer. In the latter case, however, the target is assumed to be a digital computer; the abstract model is not a dynamical system but a digital computer; and the concrete digital computer does not *simulate* but rather *realizes* the abstract system. Indeed, the abstract model is often specified *by* providing the concrete computer which realizes it. Since they are identical in computational structure, both will be relevantly similar to the target if either is; therefore, both abstract and concrete systems count as models.

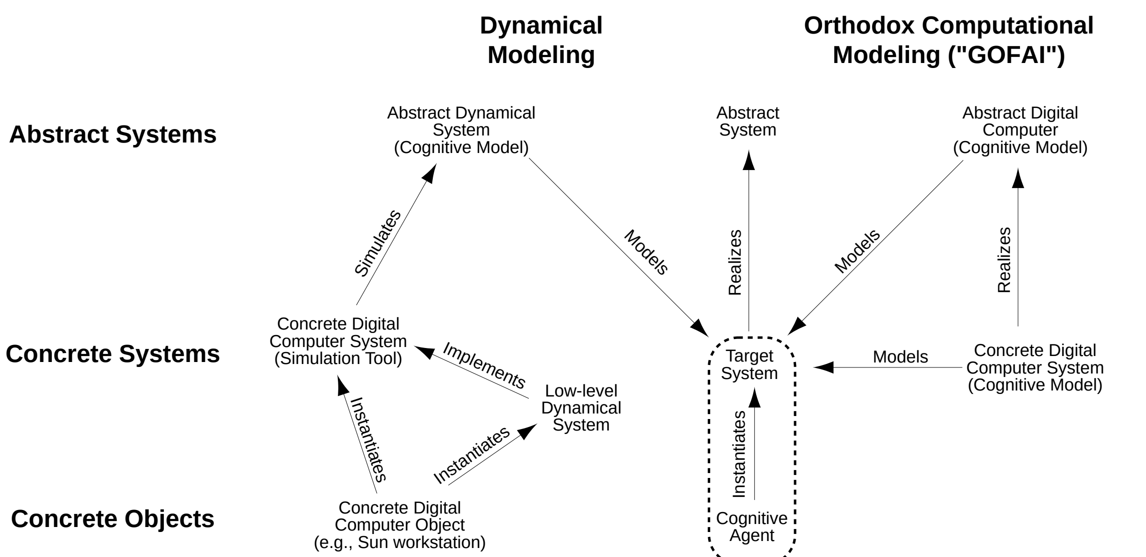

Figure 2. Basic structure of dynamical modeling as opposed to the kind of computational modeling found in mainstream cognitive science.

The basic structure of dynamical modeling is nicely illustrated by the Busemeyer & Townsend work. There are many parallels with classical mechanics. Such work comes perhaps closest to realizing the Humean dream. However, it would be misleading to suggest that dynamical modeling in cognitive science is stuck in the mold of classical physics. Obviously, cognitive phenomena differ in important ways from ordinary physical phenomena. Dynamical cognitive science has had to generate its own variations on traditional practices of dynamical modeling. Dimensions along which such variation is found include: (a) To what do model variables correspond? The quantities invoked in dynamical accounts often differ fundamentally from ordinary physical quantities. "Valence" and "preference," for example, do not appear in textbooks of mechanics. (b) At what level is the correspondence with the target? In physical models individual variables are usually taken to correspond directly to concrete physical quantities. In dynamical modeling in cognitive science, there might be *no* concrete quantity corresponding to individual variables. The correspondence between model and reality is at higher levels of dynamical structure. Individual units of a connectionist model, for example, may be significant only insofar as they support attractors which do correspond to aspects of cognition, such as a recognition state. (c) Is the correspondence quantitative or qualitative? Physical models are generally expected to match empirical data in more or less precise quantitative detail. A model of global warming, for example, should tell us exactly how much average temperature will rise. Such virtue is less common in dynamical cognitive science: as often as not, models match data qualitatively, at some level of abstraction. (In this respect dynamical modeling apes computational modeling.)

4.2.2 Dynamical Tools

Understanding cognitive agents *as* dynamical systems means more than just using certain kinds of models. Those models, and so the cognitive performances themselves, must be understood *dynamically*. Roughly, this means taking the resources of *dynamics*—as opposed, for example, to mainstream computer science—as the basic descriptive and explanatory framework. But what are those resources?

Within dynamics there is a convenient distinction between dynamical modeling, on one hand, and dynamical systems theory (DST) on the other. Dynamical modeling is a branch of applied mathematics; its concern is to understand natural phenomena by providing abstract dynamical models. The skeletal structure of such modeling was described in the previous section. The theory of dynamical modeling is a powerful repertoire of concepts, proofs, methods, etc., for use in this activity. DST, on the other hand, is a branch of pure mathematics. Its domain extends to any kind of describable change, but it focuses attention particularly on systems for which there is no known way to specify behaviors as functions of time (e.g., systems whose rule is a set of nonlinear differential equations with no solutions). The fundamental move is to conceptualize systems *geometrically*, i.e., in terms of positions, distances, regions, and paths in a space of possible states. DST aims to understand structural properties of the *flow*, i.e., the entire range of possible paths.[14]

There is no clear line between these two sides of dynamics, but the contrast is significant. Hume envisioned psychology as dynamical modeling, but that alone does not suffice. The distinctive complexities of cognition yield to scientific understanding only when dynamical modeling is enriched by the perspective and resources of DST. Poincaré pioneered DST late last century, but the bulk of it has only been developed in the last few decades. Contemporary dynamics would be a whole new subject to Newton or even Maxwell. Hume aspired to be the Newton of the mind, but in hindsight Poincaré would have made a better model.

Dynamics plays much the same role in dynamical cognitive science as computer science (the theory of computational systems, particularly digital computers) plays in traditional cognitive science. Computer science is not itself a theory of cognitive processes. Rather, it provides a powerful set of tools for use in developing accounts of particular aspects of cognition. Therein lies the hard empirical work of mainstream cognitive science. Likewise, dynamics does not somehow automatically constitute an account of cognition. It is a highly general framework which must be adapted, supplemented, fine-tuned, etc., to apply to any particular cognitive phenomenon. This typically involves merging dynamics with other

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[14] For introductions to dynamical modeling, see (Beltrami, 1987; Luenberger, 1979). For introductions to dynamical systems theory, see (Abraham & Shaw, 1982) or (Baker & Gollub, 1990). (Abraham, Abraham, & Shaw, 1992; Kelso, Ding, & Schöner, 1992; Norton, 1995) are chapter length overviews of dynamics for cognitive scientists.

constructs (e.g., the schema (Rumelhart, Smolensky, McClelland, & Hinton, 1986b)) or theoretical frameworks (e.g., ecological psychology (Turvey & Carello, 1995)). Some authors have argued for even more dramatic reorientations in our understanding of dynamical systems for the purposes of understanding biological or cognitive systems. See, for example, the work of Robert Rosen on "anticipatory systems" (Rosen, 1985) and George Kampis on "component systems" (Kampis, 1991).

Contemporary dynamics provides powerful resources for describing general properties of the behaviour of systems. These resources can be brought to bear even in the absence of an actual equation-governed model. If done rigorously, this can buy a qualitative or preliminary understanding of the phenomenon, which may be the best available and forms a solid foundation for further exploration.[15] This approach is useful in situations where, for whatever reason, providing a model is not currently feasible (e.g., Thelen, 1995).

4.2.3 Dynamical perspective

At the highest level, there are a number of general characteristics of a broadly dynamical perspective on some natural phenomenon. The following stand out particularly strongly when the subject is cognition and the contrast is with a computational approach:

4.2.3.1 *Change versus state*. Change and state are like two sides of one coin. Nevertheless, theoretical perspectives can differ in their primary emphasis or focus. Dynamicists are interested, in the first instance, in how things change; states are the medium of change, and have little intrinsic interest. Computationalists, by contrast, focus primarily on states; change is just what takes you from one state to another.

4.2.3.2 *Geometry versus structure*. How are states of a system conceptualised? Computationalists focus on internal structure, and in particular on internal combinatorial or syntactic structure—how basic pieces are combined to form structured wholes. Dynamicists, by contrast, understand a state geometrically, in terms of its *position* with respect to other states and features of the system's dynamical landscape such as basins of attraction. In other words, they focus on where the state is, rather than what it is made up of.

4.2.3.3 *Structure in time.* Sophisticated cognition demands structural complexity in the cognitive system. How is that structure realized? Computationalists tend to think of it as laid out statically—as all present at one time—and of cognition as simple transformations of static structures. DST suggests an alternative. Systems with simple states—perhaps just one variable—can behave in very complex ways. This enables dynamicists to think of cognitive structure as laid out temporally, much like speech as opposed to the written word. Cognition is then seen as the simultaneous, mutually influencing unfolding of complex temporal structures.

4.2.3.4 *Timing versus order*. Dynamicists tend to be interested in how behaviors happen in time, whereas computationalists are interested in what the behaviour is, regardless of timing details. Put another way, computationalists focus on *which* states the system passes through, whereas dynamicists focus relatively more on *when* it passes through them.

4.2.3.5 *Parallel versus serial*. Dynamicists tend to think of systems as operating in parallel, i.e., all aspects changing interdependently at the same time. Computationalists, by contrast, tend to think of systems as serial: most variables remain unchanged in any given state transition. For a dynamicist, change is standardly global; for a computationalist, change is standardly local.

4.2.3.6 *Ongoing versus Input/Output*. Computationalists standardly think of a process as commencing with an input to the system. The task for the system is to produce an appropriate output, and it does so via a sequence of internal operations culminating in the system halting with that output. Dynamicists, by contrast, think of processes as always ongoing, not starting anywhere and not finishing anywhere. The goal is not to map an input at one time onto an

[15] If done poorly, on the other hand, it is little more than handwaving with impotent metaphors. The jargon of dynamics does, unfortunately, provide all too many opportunities for pseudo-scientific masquerading.

output at some later time, but to constantly maintain appropriate change.

4.2.3.7 *Interaction: state-setting or coupling?* How does a cognitive system interact with other things, such as the environment? Computationalists standardly think of interaction as setting *state*; the system changes in its own way from that state, until new input resets state again. Dynamicists recognize an alternative: interaction can be a matter of parameters influencing the *shape* of change. Input is conceived as an ongoing influence on the direction of change, and output as ongoing influence on something else, just as a radio set is continuously modified by an incoming signal and at the same time is delivering its sound. Sometimes interaction is a matter of coupling—two systems simultaneously shaping each other's change.

4.2.3.8 *Representations.* Standard explanations of how systems come to exhibit sophisticated cognitive performances advert to internal representations. Computationalists take representations to be static configurations of symbol tokens. Dynamicists conceive representations very differently. They find their representations among the kinds of entities that figure in DST, including parameter settings, system states, attractors, trajectories, or even aspects of bifurcation structures (e.g., Petitot, 1985a). Currently, most dynamicists make use of only the tip of the theoretical iceberg that is dynamics. As dynamical modeling increases in mathematical sophistication, we can expect representations to take even more exotic forms.

4.2.3.9 *Anti-representationalism.* Unlike digital computers, dynamical systems are not inherently representational. A small but influential contingent of dynamicists have found the notion of representation to be dispensable or even a hindrance for their particular purposes. Dynamics forms a powerful framework for developing models of cognition which sidestep representation altogether. The assumption that cognition must involve representations is based in part on inability to imagine how any non-representational system could possibly exhibit cognitive performances. Within the dynamical approach, such systems can be not only imagined, they can be modelled and constructed (see, e.g., Beer, 1995a; Beer, 1995b; Freeman & Skarda, 1990; Harvey, 1992; Husbands, Harvey & Cliff, 1995; Skarda & Freeman, 1987; Wheeler, 1994).

4.3 The Dynamical Hypothesis, Exposed.

Summarizing these points yields the following compact formulation of the DH: For every kind of cognitive performance exhibited by a natural cognitive agent, there is some quantitative system instantiated by the agent at the highest relevant level of causal organization, such that performances of that kind are behaviors of that system; in addition, causal organization can and should be understood by producing dynamical models, using the theoretical resources of dynamics, and adopting a broadly dynamical perspective.

5. Considerations Favoring the Dynamical Hypothesis

What can be said in favor of the DH? Specific aspects of cognition generate idiosyncratic cases for dynamical treatment, but our interest here is in general considerations. Space limits preclude complete coverage, but the following arguments are among the most important.[16]

Most obviously, there is a kind of empirical success argument, paralleling Newell and Simon's primary argument for the CH.[17] It starts from the impressive track record of dynamics itself. Dynamics is arguably the most widely used and powerful explanatory framework in science. An extraordinary range of natural phenomena have turned out to be best described as—i.e., to *be*—a matter of interdependent coevolution of quantitative variables. It would hardly be surprising if dynamics found application in the study of cognition as well. Michael Turvey for one has long been arguing that the proper road to a deep understanding of natural cognition is to strive patiently to extend and apply the tried

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[16] Discussion of a wider range of considerations is found in (van Gelder & Port, 1995).

[17] In their celebrated paper "Computer Science as Empirical Enquiry," Newell and Simon argue for the computational hypothesis primarily on the basis of the success of AI in producing intelligent computers, and the success of computational cognitive science in modeling cognition. The only other argument they mention is "the absence of specific competing hypotheses." See (Newell & Simon, 1976).

and true techniques of natural science to incrementally more complex biological and cognitive phenomena (see, e.g., Swenson & Turvey, 1991; Turvey & Carello, 1981).

The empirical success argument in the form just presented has little weight on its own, for cognition differs from other phenomena in important ways. Its force really comes into play when combined with evidence of success in cognitive science itself. There is now a considerable amount of such evidence, some of which has already been cited. Of course, the claim is not that there is *now* sufficient empirical evidence to establish the supremacy of the DH. Indeed, there are numerous aspects of cognition for which, considered in isolation, the case for dynamical treatment is currently weak at best. The argument is that such successes as do exist, in conjunction with the general track record of dynamics, augurs well for the DH. The two levels of the argument require and reinforce each other.

What *explains* any success the dynamical approach has exhibited thus far? And what underpins confidence that there will be more? The foremost consideration is simply that *natural cognition happens in real time*. This blunt fact is multi-faceted. Every cognitive process unfolds in continuous time, and the fine temporal detail calls out for scientific accounting. Moreover, many cognitive structures are *essentially* temporal: like utterances, they exist only as change in time. Often, getting the timing right is critical to the success of cognitive performance; this is especially so when in direct interaction with surrounding events.

Hobbesian computational models have made a bet that cognitive phenomena can be described in a way that abstracts away from the full richness of real time, replacing it with discrete orderings over formal states. From a dynamical perspective, this looks ill-advised. Dynamics, by contrast, takes the nature of change in time as its primary focus. It is the preeminent mathematical framework for description of temporal phenomena. Taking cognitive agents to be dynamical systems allows scientific explanation to tap into this power.

A third argument focuses on the *embeddedness* of cognition. Even the loftiest forms of natural cognition are in fact embedded three times over: in a nervous system, in a body, and in an environment. Any account of cognition must eventually explain how it is that cognition relates to that which grounds and surrounds it. Now, suppose the behavior of brain, body and environment all turn out to be best described in dynamical terms. Suppose, in short, that cognition is thoroughly embedded in dynamics. The challenge would then be to explain how cognitive phenomena are constituted of, shaped by and interact with those dynamical phenomena. While explaining embeddedness is never trivial, it stands to reason that there will be greater problems in relating systems of fundamentally different kinds than in relating systems of fundamentally the same kind. Mainstream computational cognitive science has for the most part simply shelved problems of embeddedness, preferring to study cognition independently of its neurobiological realization, and treating the body and environment as belonging on the far side of occasional symbolic inputs and outputs. When embeddedness is confronted head-on, dynamical accounts of cognition immediately become attractive. For example, one virtue of the Catherine Browman and Louis Goldstein dynamical *phonology* (Browman & Goldstein, 1992) is that it integrates directly with Elliot Saltzman's dynamical model of *speech coordination* (Saltzman & Munhall, 1989). Dynamical cognition sits comfortably in a dynamical world.

A fourth argument focuses on the *emergence* and *stability* of cognition. Investigation of some complex phenomenon can always take at least two directions: what is it like? And, how does it get—and stay—that way? In the case of cognitive mechanisms and processes, we can address their nature, or how it is that they arise and are sustained. In the long run, our answers to these questions must hang together. Natural cognitive agents exhibit extraordinary levels of structural complexity, yet there are no architects or engineers responsible for building and maintaining that structure. The generic name for the answer to the problem of the emergence and stability of cognition is *self-organization*. Self-organization of interesting kinds of complex order appears to require systems in which there is simultaneous, mutually constraining interaction between large numbers of components. DST is the dominant mathematical framework for describing the behavior of such systems. In short, the claim is that we must understand cognitive agents as dynamical systems, since only in that way will our account of what cognition is be properly integrated with our account of how the world sustains any of it.

Each of these lines of thought was cast in the form of an attempt to demonstrate that the DH is basically *true*. With respect to that goal, they are obviously not “knock down” arguments. They do, however, indicate that the hypothesis is worthy of sustained empirical investigation of precisely the kind that has been and is being conducted, and which forms the basis of the formulation of the DH presented here.

6. The General Objections

This section considers a selection of general objections to the DH as an open empirical hypothesis. As John Stuart Mill said, “three-fourths of the arguments for every disputed opinion consist in dispelling the appearances which favour some opinion different from it.”[18] Addressing these objections is also a useful way to elaborate and clarify the hypothesis.

The objections considered fall into two main categories: those purporting to show that the DH is not a genuine *alternative* to the CH, and those purporting to show that it is not *open*, i.e., its empirical inadequacy is somehow already determined. All amount to sweeping attempts to dismiss or downplay the DH in advance of detailed empirical investigation. All mix insight with confusion to produce plausible but misguided attacks.

6.1 The “Trivially True” Objection

> *Everything is a dynamical system. Cognitive agents must be dynamical systems at some level. The DH is trivially true, and makes no substantial claim about the nature of cognition.*

This objection is mostly bluff. No doubt there is *some* vague sense in which it could be said that everything is a dynamical system. Properly interpreted, however, the DH makes a much more specific claim.

On one hand, according to the nature hypothesis, cognitive agents instantiate quantitative systems at the highest relevant level of causal organization. It may be trivial that every cognitive agent instantiates some dynamical system or other. It is certainly not trivial that every cognitive performance is at the highest level a dynamical phenomenon. This is not true of ordinary digital computers, and according to the orthodox CH, it is not true of people.

On the other hand, according to the knowledge hypothesis, cognition can be *understood* in dynamical terms. If *this* were trivially true, cognitive science would have been completed long ago. In practice, it is very challenging to establish that some aspect of cognition can be understood dynamically. Patient steps in this direction are the stuff of which whole careers are made. Some of the greatest achievements in science have amounted to describing some natural phenomenon (e.g., celestial motion) in dynamical terms. This activity is no more trivial in cognitive science than anywhere else.

6.2 The “False Opposition” Objection 1 - Computers are Dynamical Systems

> *Ordinary electronic computers are dynamical systems. In general, digital computers are dynamical systems as well. The DH is therefore not an interesting alternative to the CH.*

This objection gains plausibility by mixing together at least three distinct lines of thought. Each is based on a different reason for thinking that digital computers are dynamical systems. Each has elements of truth but also problems.

6.2.1 Digital computers are state-determined, rule-governed, etc.

A first line of thought takes digital computers to count as dynamical systems because they satisfy some broad definition; e.g., they are state-determined systems, or they are governed by some mapping, etc.. This kind of move is reasonable in the light of some strands of contemporary usage (see Table 1). However, it only appears to constitute an objection to the DH because it equivocates on the term “dynamical system”. The DH takes cognitive agents

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[18] (Mill, 1975) Chapter 2. In “Computing Machinery and Intelligence” (Turing, 1950), Turing rebuts nine objections to his stance on whether computers can think; most are not attributed to anyone in particular. This paper follows these august precedents. Except where noted, the objections are not known to have appeared in print; rather, they are based on the author’s experience of reactions to the dynamical hypothesis when expounded in public presentations or in related work.

to be dynamical systems in a much more specific sense, i.e., quantitative systems.

6.2.2 Digital computers are quantitative systems.

A second line of thought does not equivocate. Rather, it suggests that the definition of dynamical systems as quantitative systems is broad enough to embrace digital computers as such.

Digital computers and dynamical systems are two classes of systems picked out by reference to different properties: roughly, effectiveness and interpretation as opposed to quantitativeness. Generally, systems exhibiting the one property fail to exhibit the other and vice versa. In a typical Turing Machine, for example, there is no systematic relationship between system behavior and distances between states. A tape square's values are *different* but not relevantly *distant* from each other. System behavior turns only on which values happen to obtain (i.e., type identity), not on how far those values are from any others. Similarly in the case of time. Turing Machine states are indexed by means of the positive integers. There are distances between integers, but these distances generally bear no systematic relationship to system behavior. The integers might just as well be replaced by any other sufficiently large merely ordered set, such as names in the New York telephone directory.

Since there are generally no relevant distances in state or time in digital computers, it makes no sense to describe their behavior in terms of rates of change (not to mention dependence on rates of change). This is why in practice computer scientists don't bother with distances between states, rates of change, etc..

There is a common temptation to suppose that digital computers count as quantitative systems arising from the correct observation that certain metrics apply to *any* set of values, regardless of the nature of those values (e.g., (Padulo & Arbib, 1974), pp.91-2). Thus *every* variable is a quantity, and so even digital computers have metric spaces as state sets. The crucial point, however, is that the distances measured by these trivial metrics bear no systematic relationship to system behavior. Turing Machines bounce around their state spaces in ways which will seem utterly erratic until one realizes that their order is based on formal properties, not quantitative properties.

Oranges come in many kinds. Some are valencia, some are expensive; occasionally, an orange is both. Similarly with digital computers and dynamical systems. In coincidental, contrived, or trivial cases, one and the same set of variables might satisfy the conditions for both classes. Nevertheless, digital computers and dynamical systems are classes of systems picked out by reference to fundamentally different properties. In general, systems exhibiting one property fail to exhibit the other.

6.2.3 Digital computers are dynamical systems at the hardware level.

A third line of thought is based on the idea that all *concrete* digital computers are in fact dynamical systems at some lower level of description. For example, standard general purpose digital computers such as Macintoshes are dynamical systems at the level of electronic circuits. Now, there is truth in this, but not enough to vitiate the relevant contrast. The fundamental problem here is that "are" is too crude; it rides roughshod over a number of issues.

To sort out the relationship between digital computers and lower-level dynamical systems, we must distinguish at least three different relationships: instantiation, identity, and implementation. At any given time a Macintosh *instantiates* a great many different systems at different levels. One of these is the high-level digital computer by virtue of which, for example, it calculates my taxes. Presumably it also instantiates some hugely complex electrical dynamical system. The Macintosh is not *identical* with either of these systems. Neither are they strictly identical with each other; most obviously, they have different numbers of variables. Of course, the macrovariables of the high-level digital computer are ultimately built up out of the microvariables of the electronic system, and so there is presumably some lower-level dynamical system *implementing* the high-level digital computer. Thus, while there is one clear sense in which the digital computer "is" some lower-level dynamical system, there is also a clear sense in which it "is" not that system.

6.3 The "False Opposition" Objection 2 - Dynamical Systems are Computers

Much recent research in computation theory has been exploring the computational power

> *of dynamical systems. There is no inherent conflict between dynamics and computation, and so there is no real opposition between the computational and dynamical hypotheses.*

It is true that there is no inherent conflict between dynamics and computation, but the conclusion does not follow. Again, the issues must be teased out more carefully.

Recall from Section 3.2 that effective computation is a specific kind of computation, resulting from a certain kind of constraint on the processes involved. Other kinds of computation result from adopting different constraints. In particular, we can focus attention on some class of dynamical systems (Blum, Cucker, Shub, & Smale, forthcoming; Blum, Shub, & Smale, 1989; Moore, 1991; Moore, 1996). As long as there is some way to specify the "questions" and "answers" we can see dynamical processes as computing functions. For example, Hava Siegelmann has extensively studied the computational properties of one class of dynamical systems, recurrent neural networks (Siegelmann & Sontag, 1994). Indeed, it can be proved that certain classes of dynamical systems are *more* powerful—can compute a wider class of functions—than Turing Machines.[19] So, dynamical systems can compute, i.e., be computers, without needing to be digital computers. This is why research into the power of dynamical systems an interesting new branch of computation theory!

The most famous and influential of all critiques of the mainstream computational approach to cognition is surely *What Computers **Still** Can't Do* (Dreyfus, 1992). In that book, Dreyfus noted that brains might well be turn out to be "analogue" rather than digital computers. Similarly, as Churchland and Sejnowski have argued at length, biological neural networks can be understood as computing in ways that differ fundamentally from ordinary digital computation (Churchland & Sejnowski, 1992). Like these perspectives, the DH can embrace the idea that cognitive processes are computational, while preserving a contrast with the CH. This does not diminish but rather fortifies the DH, by allowing it incorporate computational ideas without inheriting orthodoxy's excess baggage.

6.4 The "False Opposition" Objection 3 - Dynamical Systems are Computable

> *There is no good reason to think that any cognitive process is not effectively computable. Even if cognitive agents are dynamical systems, they will still be computable systems. Therefore, it is misguided to present the DH as an alternative to the CH.*

One particularly troublesome mistake is blurring the distinction between *computational* and *computable*. Just as employers and employees stand at opposite ends of an employment contract, so *computational* and *computable* stand at opposite ends of the relation *computes*. The former applies to whatever *does* the computing; the latter to whatever *gets* computed. In classical theory, a digital computer does the computing, and a function over the integers gets computed. The effectively computable functions over the integers are all and only the partial recursive functions.

Computation theorists, including Turing himself, quickly turned to asking what *else* might be effectively computed. Via arbitrarily good approximation, the purview of effective computation was gradually extended to embrace real numbers, functions over real numbers, differential equations, and so on (Earman, 1986; Grzegorczyk, 1957; Turing, 1936). In this way, issues of effective computability can be raised for all the standard mathematical constructs of analysis and physics. Just what is and is not effectively computable rapidly becomes a rather complicated business (see, for example, Pour-El & Richards, 1989).

Now, we can regard a *system* as computable just in case its behavior is governed by some computable function. The solar system of classical mechanics is effectively computable in this sense. Currently, as far as we can now see, most if not all dynamical systems of practical relevance to cognitive science are effectively computable.[20] This doesn't make those systems

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[19] The general result that dynamical systems can have "super-Turing" capacities need not be very surprising. Digital computers are a strictly delimited class of systems, and it makes sense that classes defined by alternative sets of constraints would allow more powerful processes.

[20] Note that *effectively computable* is a theoretical notion; it is not the same as *computable in practice*. As chaos theory reminds us, some systems will always outstrip our finite computing resources.

digital computers. Digital computers can compute functions governing systems which are not themselves digital computers. Thus, the computability of dynamical models does not destroy the contrast between the dynamical and computational hypotheses.

6.5 The "Straw Man" Objection

Turing Machines are caricatures of computers. The DH is being matched against a straw man. It is not a substantial alternative to the CH as properly understood.

There are two issues here. One is whether the CH, as characterized here, is a straw man. Two considerations suffice to dispel this objection. First, the characterization offered here is just the standard philosophical account, as developed in numerous places.[21] Second, a great many models in cognitive science (e.g., those developed within the SOAR (Newell, 1991) framework) do in fact conform to that account.

The other issue is whether the standard account misunderstands the "true" CH, i.e., deeply misconceives computers and computational modeling in cognitive science. This may be; Brian Smith, for one, has begun formulating a critique of received wisdom in this area (Smith, 1996; Smith, forthcoming). These issues go beyond the scope of the present discussion. If and when some superior understanding of the CH clearly supplants the orthodox account, the relationship between the dynamical and computational hypotheses will need to be reconsidered.

6.6 The "Description, Not Explanation" Objection

Dynamical models are at best descriptions of the data, and do not explain why the data take the form they do. For genuine explanation, we need computational models describing the underlying causal mechanisms.

Dynamical theories of cognitive processes are deeply akin to dynamical accounts of other natural phenomena such as celestial motion. Those theories constitute paradigm examples of scientific explanation. Consequently, there is no reason to regard dynamical accounts of cognition as somehow explanatorily defective.

Dynamical explanations typically proceed by providing equations defining an abstract model. Many factors are relevant to the goodness of a dynamical explanation, but it should at least capture succinctly the relations of dependency, and make testable predictions. A *poor* dynamical account may amount to little more than *ad hoc* "curve fitting", and would indeed count as mere description. It's problem, however, is that it is poor, not that it is dynamical.

Traditional computational cognitive science offers explanations of a quite distinctive kind (Haugeland, 1978), and many cognitive scientists have become so accustomed to such explanations that anything else seems inadequate. The explanations offered in dynamical cognitive science are indeed quite different (Garson, 1996; van Gelder, 1991), but are not for that reason inferior.

6.7 The "Not As Cognitive" Objection

Dynamics is a general purpose framework which applies to any behavior of an agent, regardless of whether that behavior is cognitive or not. Dynamics does not focus on the specifically <u>cognitive</u> aspects of systems; it does not explain cognitive performances "as cognitive." Genuine explanation in cognitive science must be framed in terms of aspects of cognitive agents other than their purely dynamical properties.

This objection concedes that dynamical explanations are nontrivial empirical explanations, and that they really are quite different from computational explanations. It challenges the nature of the explanation being offered. Dynamics is held to be too general, failing to explain cognition in terms of its distinctive features.

Underlying this objection is an important misconception about the DH. That hypothesis asserts that cognitive agents are dynamical systems of quite special kinds. Therefore, as emphasized in Section 4.2.2, understanding cognitive agents *as* dynamical systems is not simply the routine application of generic dynamics to systems that happen to be exhibiting cognitive performances. It requires that the resources of dynamics be developed and

[21] See note 7.

supplemented in order to provide explanations of those special kinds of behaviors. Thus, dynamical cognitive science always incorporates considerations distinctive to particular kinds of cognition into dynamical frameworks to produce explanations that are fundamentally dynamical in form, but are nevertheless tailored to explain cognitive performances "as cognitive." To take just one example, Jean Petitot merges Ron Langacker's cognitive grammar with René Thom's morphodynamics to yield a thoroughly dynamical approach to syntax (Petitot, 1995).

6.8 The "Wrong Level" Objection

There is an important role for dynamical descriptions in any complete account of the nature of a cognitive agent, but they are pitched too low to explain cognition.[22]

A common misconception about the dynamical approach is that it operates solely or primarily at "lower" or "micro" levels of description. In fact, dynamics is not intrinsically limited to *any* level or domain. In the natural sciences, dynamics finds application at all levels from quantum mechanics to cosmology. It gets its grip wherever sets of interdependently changing quantities are found. Similarly in cognitive science: dynamicists develop their explanations at the level of theoretical interest, whatever that might be (see Section 4.1).

One significant difference between the dynamical approach and PDP-style connectionism turns on this point. They agree that cognitive performances are behaviors of dynamical systems. The PDP approach, however, takes those systems to be high-dimensional neural networks operating at a level below that of orthodox descriptions (Smolensky, 1988); as expressed in the titles of the famous volumes,[23] they constitute the *microstructure* of cognition. The dynamical approach is more catholic; it embraces dynamical models of all kinds and at all levels.

6.9 The Structure Objection

Sophisticated cognitive performances require complex internal structures. The dynamical approach is taking a huge step backwards in trying to replace symbolic representations with quantities. To explain high level cognition, dynamical systems will have to implement computational mechanisms.

Almost everyone now agrees that most kinds of cognitive performance can only be explained by reference to complex structures internal to the system responsible for those performances. Still, it remains an open question what form those structures might take. Hobbesian cognitive scientists are banking on the idea that they are the kind of structures found in digital computers, i.e., symbol structures (Newell & Simon, 1976) or "classical" combinatorial representations (Fodor & Pylyshyn, 1988). Lying behind this idea is an assumption that the kinds of complex structures required cannot exist in any system except by instantiating digital symbol structures.

However, as dynamical cognitive science has matured, it has become apparent that dynamical systems can incorporate combinatorial structures in various ways without merely implementing their digital cousins (van Gelder, 1990). For example, arbitrarily many structures can be mapped onto states of a dynamical system, such that these states can then be used as the basis of systematic processing (e.g., (Chrisman, 1991; Pollack, 1990)). Other work has found combinatorial structure in the attractor basins of appropriate dynamical systems (Noelle & Cottrell, 1996), or in the trajectories induced by sequences of bifurcations ("attractor chaining", (van Gelder & Port, 1994)). The possibilities have really only begun to be explored. The dynamical approach is not vainly attempting to do *without* complex internal structures. Rather, it is in the process of dramatically reconceiving how they might be instantiated.

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[22] The "Peripheral" objection is very similar, and is dealt with by a similar response. It maintains that dynamical explanations are concerned with peripheral aspects of cognitive agents rather than cognition itself, which is more "central."

[23] (McClelland, Rumelhart, & The PDP Research Group, 1986; Rumelhart, McClelland, & The PDP Research Group, 1986a).

6.10 The Complexity Objection

Natural languages are only effectively described by some form of context-sensitive grammar. In the standard Chomskian hierarchy, languages of this complexity can only be handled by computers at least as powerful as linear-bounded automata (LBAs). Therefore, natural language speakers must be computers at least as powerful as LBAs.

The conclusion of this argument is ambiguous, between computers in general and digital computers. On the former interpretation, the argument is sound, but fails to conflict with the DH. It was pointed out above that dynamical systems can compute, i.e., be computers. The complexity of natural language constrains speakers' computational *power*, but not the *kind* of computer they instantiate. It remains an open empirical question whether the computers in question are best thought of as digital or dynamical (Elman, 1995).

On the latter interpretation, the argument simply equivocates. The premises establish that speakers must be computers in *some* sense; the conclusion claims they must be digital computers. The dominance of digital computers in the theory of computation, cognitive science, and computer technology, has created an unfortunate tendency to confuse computers in general with digital computers. This is what drives the objection.

6.11 The "Not Cybernetics Again!" Objection

The dynamical approach is just cybernetics returning from the dead.

What was cybernetics? Wiener famously defined it as "the science of communication and control in man and machine" but it soon developed into an even wider enterprise: a kind of general, non-reductionistic study of *systems*, particularly self-sustaining systems in their environments (see, e.g., (Parsegian, 1973)). Throughout its brief ascendancy, cybernetics enthusiastically embraced anything of conceivable relevance to complex systems, including information theory, communication theory, automata theory, neurophysiology, systems theory, game theory and control theory.

Dynamics was certainly mixed up in all this, and the DH is sometimes traced back to a leading cyberneticist, H. Ross Ashby. Still, the demise of cybernetics implies little about the contemporary dynamical approach, for they differ in important ways. The DH is, by comparison, tightly circumscribed. It is concerned with cognition specifically, rather than systems generally, and is defined in terms of a core commitment to a single framework. The fate of cybernetics as a whole no more attaches to the dynamical approach than it does to other disciplines with ancestral links to cybernetics, such as computational neuroscience and artificial intelligence. Moreover, much more powerful tools are available today. The bulk of DST has been developed in the period *since* cybernetics. Also, dynamicists now have on their desks computer simulation tools (hardware and software) beyond the dreams of cyberneticists. Where cyberneticists could only speculate, dynamicists can now furnish and understand complex models.

6.12 The "Humans Compute" Objection

Humans can do arithmetic in their heads. At least some cognitive activity is specifically <u>digital</u> computation. Therefore, the DH cannot be the whole truth about cognition.

If it is granted that mental arithmetic and like processes are, literally, digital symbol manipulation inside the head, then the DH should indeed graciously concede. The *general* truth of the DH is compatible with certain special activities counting as exceptions. However, we should be wary of granting, in advance, that mental arithmetic *is* symbol manipulation. Certainly, it *seems* like symbol manipulation: numerals, lines, etc. are "seen in the mind's eye". It does not follow that there are symbols in the head, i.e., that the states and processes that subserve such "seeing" actually instantiate symbols and their manipulations. Imagining the Eiffel Tower does not entail that one has the Eiffel Tower, or even a picture of it, inside one's head (Ryle, 1984, Ch.8). We must not confuse the content of experience with the mechanisms implementing it. As usual, the question turns out to be the empirical one: in the long run, what kind of models provide the best account of the mechanisms underlying the relevant kind of cognitive performance?

7. Conclusion

The contemporary dynamical approach to cognition is part of a much wider scientific trend.

In recent decades, there have been dramatic developments in the mathematics of DST, especially the theory of nonlinear systems, complexity, and chaos. At the same time, there has been exponential growth in available computing power, and the arrival of sophisticated programs for exploring dynamical systems. The result is that dynamical theorising has come to be applied to a wide range of natural phenomena that were previously either ignored entirely, or regarded as beyond the scope of standard forms of scientific explanation. So with cognition. The Humean dream of a dynamics of cognition can now be seriously pursued. The explanatory umbrella which covers so much of the rest of the natural world so effectively is gradually being extended to cover cognition as well.

The DH encapsulates the core commitment of the emerging dynamical approach. This target article has attempted to say what it means, and to establish its status as an open empirical hypothesis standing as a substantial alternative to the CH. It has not attempted to demonstrate that cognitive agents *are in fact* dynamical systems. There is mounting evidence that certain aspects of cognition are best thought of dynamically, but many others remain completely unaddressed. Only sustained empirical investigation will determine the extent to which the DH—as opposed to the CH, or perhaps some other hypothesis entirely—captures the truth about cognition.

APPENDIX

Table 2. *Key terms and their meanings in the present discussion. This table has no pretensions beyond partially summarizing the particular regimentation proposed in this paper for the purpose of clarifying the DH in cognitive science.*

Term	**Meaning in this paper**
Variable	Anything that changes over time..
System	A set of variables changing interdependently.
Instantiation	A relation between a concrete system and some object or part of the world. An object instantiates a system when all the variables of the system are features of the object.
Implementation	A relation between concrete systems, obtaining when the variables of one system are somehow built up out of the variables of the other.
Parameter	Something outside (i.e., not a member of) a system, but upon which change in the system depends.
Coupling	Mutual direct dependence. Variables *x* and *y* are coupled when the state of *x* shapes change in *y* and vice versa.
Concrete system	A system whose variables are all concrete features of the concrete world changing in real time.
Abstract system	A system whose variables are all abstract entities.
Realization	A relation between a concrete system and an abstract one, obtaining when the former has the same structure as the latter.
Time	Any intrinsically ordered set, serving to provide orderings over other things. Real time is the set of instants at which things can happen, ordered by priority (before/after).
Computer	Anything that computes (carries out computation).
Computation/ Computing	Transforming some kind of question (e.g., input object or start state) into some kind of answer (e.g., output object or final state).
Computational	Anything that computes (carries out computation).

Digital Computer	A computer carrying out effective computation over representations. A digital computer must have digital variables, discrete time, algorithmically governed behavior, and an interpretation.
Effective	Succeeding in a finite number of basic operations governed by an algorithm.
Computable	Capable of being computed; alternatively, being governed by a computable function.
Quantity	A variable with a metric over its values.
Dynamical System	A quantitative system. A system that is at least quantitative in state; may also be interdependently quantitative in state and time, or even rate dependent.
Identity	"being the very same thing as". Identity is governed by Leibniz Law: identical things have all and only the same properties. Identity for sets—and hence for systems—is having all and only the same variables.
Simulate	Compute a function approximately describing some process.
Dynamics	Two closely related kinds of mathematics, dynamical modeling and DST.
Dynamical Hypothesis (DH)	Cognitive agents are dynamical systems. See Section 4.3.
Computational Hypothesis (CH)	Cognitive agents are digital computers.

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