On bipolarity in argumentation frameworks

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Abstract

In this chapter, we propose a survey of the use of bipolarity in argumentation frameworks. On the one hand, the notion of bipolarity relies on the presence of two kinds of entities which have a diametrically opposed nature and which represent repellent forces (a *positive* entity and a *negative* entity). The notion exists in various domains (for example with the representation of preferences in artificial intelligence, or in cognitive psychology).

On the other hand, argumentation process is a promising approach for reasoning, based on the construction and the comparison of arguments. It follows five steps: building the arguments and the interactions between them, valuating the arguments and accounting for their interactions or not, finally determining the acceptability of the arguments and using it in order to draw a conclusion/choose a decision.

Using the nomenclature proposed by Dubois and Prade, this paper shows on various applications, and with some formal definitions, that bipolarity appears in argumentation (in some cases if not always) and can be used in each step of this process under different forms.

1 Introduction

A rational agent can express claims and judgements, aiming at reaching a decision, a conclusion, or informing, convincing, negotiating with other agents. Pertinent information may be insufficient or, on the contrary, there may be too much relevant, but partially incoherent, information. And, in case of multi-agent interaction, conflicts of interest are inevitable. So, agents can be assisted by argumentation.

Argumentation is a promising approach for reasoning with inconsistent knowledge, based on the construction and the comparison of arguments. It may also be considered as a different method for handling uncertainty [Coh85, KAEF95, Pol01]. A basic idea behind argumentation is that it should be possible to say more about the certainty of a particular fact than just assessing a certainty degree in [0, 1]. In particular, it should be possible to assess the reason why a fact holds, under the form of arguments, and combine these arguments for the certainty evaluation. Indeed, the process of combination may be viewed as a kind of reasoning about the arguments in order to determine the most acceptable among them.

Argumentation has been applied in various domains and applications such decision making and negotiation (see [Dun95, FP97, Par97, AMP00a, PSJ98, KP01, GK97, Ver02]). For example, recent works on negotiation [AMP00b, AMP00a, AP04b, KSE98, PSJ98, RRJ⁺04, RJS03, RSD03] have argued that argumentation can play a key role in finding a compromise. Indeed, an offer supported by a 'good argument' has a better chance to be accepted by another agent. Argumentation may also lead an agent to change its goals and finally may constrain an agent to respond in a particular way.

In all the disparate cases, an argumentation process follows five steps: i) constructing arguments, ii) defining the different interactions between those arguments, iii) valuating each argument, iv) selecting the most acceptable arguments and finally v) concluding. In most existing argumentation frameworks, only one kind of interaction is considered between arguments. It is the so-called defeasibility relation. However, recent studies on argumentation [KP01, Ver02, ACLS04] have shown that another kind or interaction may exist between the arguments. Indeed, an argument can defeat another argument, but it can also support another one. This suggests a notion of bipolarity, *i.e.* the existence of two independent kinds of information which have a diametrically opposed nature and which represent repellent forces.

Bipolarity has been widely studied is different domains such as knowledge and preference representation [Bou94, TP94, LVW02, BDKP02]. Indeed, in [BDKP02] two kinds of preferences are distinguished: the *positive* preferences representing what the agent really wants, and the *negative* ones referring to what the agent rejects. This distinction has been supported by studies in cognitive psychology which have shown that the two kinds of preferences are completely independent and are processed separately in the mind. Another application where bipolarity is largely used is that of decision making. In [ABP05, DF05], it has been argued that when making decision, one generally takes into account information in favour of the decisions and other information against those decisions.

In [DP06], a nomenclature of three types of bipolarity has been proposed using particular characteristics like *exclusivity* (can a piece of information be at the same time positive and negative), *duality* (can negative information be computed using positive information), *exhaustivity* (can information be neither positive, nor negative), computation of positive and negative information on the same data, computation of positive and negative information with the same process, existence of a consistency constraint between positive and negative information.

The first type of bipolarity proposed by [DP06] (*symmetric univariate bipolarity*) expresses the fact that the negative feature is a reflection of the positive feature (so, they are mutually exclusive and a single bipolar univariate scale is enough for representing them).

The second one (*dual bivariate bipolarity*) expresses the fact that we need two separate scales in order to represent both features, although they stem from the same data (so, an information can be positive and negative at the same time and there is no exclusivity). However a duality must

exist between both features.

And the third one (*heterogeneous bipolarity*) expresses the fact that both features do not stem from the same data though there is some minimal consistency requirement between both features.

The purpose of this paper is to present a comprehensive *study* on the use of bipolarity in a particular domain: argumentation frameworks. We show that bipolarity appears in the five steps of an argumentation process under different forms, and we restate bipolarity in argumentation in the nomenclature proposed in [DP06].

The paper is organized as follows: Section 2 introduces the basics of an argumentation process as well as the abstract framework proposed in [Dun95]. Sections 3, 4, 5, 6 and 7 study the bipolarity at each level of an argumentation process. Section 8 is devoted to some concluding remarks.

2 Background on argumentation systems

An argumentation process follows the five following steps:



- 1. defining the *arguments*: the notion of argument commonly refers to the concepts of explanation, proof, justification. Arguments aim to support beliefs, or to criticize an agent in order to behave in a certain way. They can take the form of a piece of text or discourse, by which one tries to convince the reader that a given claim is true, or they can be seen as a logical proof of a claim. Formally, arguments are built around an underlying representation language. Different basic forms of arguments can be encountered, depending on the language and on the rules for constructing arguments.
- 2. defining the different *interactions between arguments*: arguments formed from a knowledge base cannot be considered independently. Indeed most of the arguments are in interaction.

Two kinds of interaction are encountered¹: arguments may be conflicting or on the contrary arguments may support other arguments.

- 3. valuating the arguments: the basic idea behind this valuation process is to give a weight for each argument. The different weights make it possible to compare the arguments. Different criteria can be used in order to define the weights. For example, in [AC02a], implicit and explicit priorities are considered. In [CLS03b, ACLS04], the weights are defined on the basis of the interactions between arguments.
- 4. selecting the most acceptable arguments: it is necessary to define the status of arguments on the basis of all the ways in which they interact, and of the available valuation process. As an output of the argumentation system, the best arguments must be identified. Informally, these arguments are the most acceptable, and will help win a dispute.
- 5. concluding the argumentation: the status of arguments in turn determines the status of conclusions. So, argumentation-based defeasible inference relations can be defined from the selection of acceptable arguments or sets of arguments. And it is the same mechanism in other domains (decision, negotiation, ...).

In [Dun95], Dung has proposed a general and abstract framework for argumentation in which he focuses only on the definition of the status of arguments. For that purpose, he supposes that a set of arguments is given, as well as the different conflicts between them. We briefly recall that abstract framework (it will be extended in Section 4):

 An argumentation framework is a pair <A, R> of a set A of arguments and a binary relation R on A called a *defeat relation*.

In this document, the "defeat" word is a *generic term* which can encompass different cases:

- the conclusions of two arguments are conflicting,
- the conclusion of one argument undermines a premise of the other one,
- one argument attacks another argument and the first argument is preferred to the second one,
- • •

 $A_i \mathcal{R} A_j$ means that A_i defeats A_j . An argumentation system may be represented by a directed graph whose nodes are arguments and edges represent the defeat relation.

• The notion of *defence* is defined from the notion of defeat by: an argument A_i defends A_j against B iff $B\mathcal{R}A_j$ and $A_i\mathcal{R}B$.

In Dung's framework, only the selection step of an argumentation process is taken into account². In this work, the *acceptability of an argument* depends on its membership in some sets (acceptable sets or extensions) characterized by particular properties. It is a collective acceptability. The main characteristic properties are:

- Conflict-free: a subset S of A is conflict-free iff there exist no A_i , A_j in S such that $A_i \mathcal{R} A_j$.
- Collectively defends : a subset S of \mathcal{A} collectively defends an argument A_i iff for each argument B, if $B\mathcal{R}A_i$ there exists C in S such that $C\mathcal{R}B$.

Then several semantics for acceptability have been defined by [Dun95]:

¹Even if there are no conflict and no support between two arguments, two other kinds of "interaction" can be distinguished: "mutual compatibility" (arguments are in the same relevant domain but it can be shown that neither argument is a counter-argument or a support for the other), and "null interaction" (the domains of relevance of the two arguments do not overlap).

 $^{^{2}}$ However, some notions proposed by Dung can be used in the valuation of arguments (see [CLS03b]).

- Admissible: a subset S of A is an admissible set iff S is conflict-free and S collectively defends all its elements.
- Preferred: a subset S of \mathcal{A} is a preferred extension iff S is maximal for the set inclusion among the admissible sets of \mathcal{A} .
- Stable: a subset S of A is a stable extension iff S is conflict-free and S defeats each argument which does not belong to S.
- Grounded: a subset S of A is the grounded extension iff S is the least fixed point of the characteristic function F of $\langle A, R \rangle$

 $(F: 2^{<\mathcal{A},\mathcal{R}>} \to 2^{<\mathcal{A},\mathcal{R}>} \text{ with } F(S) = \{A \text{ such that } S \text{ collectively defends } A\}).$

These semantics are used in the selection step of the argumentation process and, sometimes, they are associated with the results of the valuation step (see for example [CLS03a]). They will be used in Section 6.

The previous notions are illustrated on the following argumentation system:

Example 1



In this system, A is defended by C_1 and C_2 , and there is only one preferred extension $\{D, C_2, A\}$.

3 Bipolarity at the argument level

3.1 Building of the arguments

As said in the introduction, the basic idea behind argumentation is to construct arguments in *favour* of a conclusion and arguments *against* that conclusion (called also defeaters), then to select the acceptable ones. The role of arguments in favour of a conclusion is to support that conclusion, whereas the role of arguments against a conclusion is to attack it. Thus, the two kinds of arguments have different and opposite roles. One might then say that arguments are presented in a bipolar way since arguments in favour of a conclusion can be considered as *positive* and arguments against the conclusion as *negative* ones.

Since the two kinds of arguments play different roles, one might wonder whether they are defined and handled in the same way. In fact this depends broadly on the considered application.

Within the negotiation framework, for instance, various natures of arguments have been distinguished in [AP04a]: *instrumental arguments, explanatory arguments, threats* and *rewards*. For each nature of argument, only one definition is proposed for both supporting and attacking a conclusion. Let us consider the case of explanatory arguments. Explanations constitute the most common category of arguments. In classical argumentation-based frameworks which have been developed for inconsistency handling in knowledge bases, each conclusion is justified by arguments. They represent the reasons to believe in the conclusion. Such arguments have a deductive form. Indeed, from premises, a conclusion is entailed.

Let \mathcal{L} be a propositional language. \vdash denotes classical inference and \equiv denotes logical equivalence. Let $\mathcal{K} = \{k_j; j = 1, ..., l\}$ be a base representing the available knowledge of an agent with k_j are formulas of \mathcal{L} . **Definition 1 (Explanatory argument)** An explanatory argument is a pair $\langle H, h \rangle$ such that:

- $H \subseteq \mathcal{K}$.
- $H \vdash h$.
- *H* is consistent and minimal (for set inclusion) among the consistent sets *H* satisfying the two previous items.

H is the premises of the argument and h its conclusion.

Example 2 Let $\mathcal{K} = \{p, p \to b, p \to \neg f, b \to f\}$ where p means penguin, b means bird, f means fly. Let $H = \{p, p \to b, b \to f\}$, $H' = \{p, p \to \neg f\}$ be two subsets of \mathcal{K} . The fact that p flies is justified by the explanatory argument $\langle H, f \rangle$. However, the conclusion f has a counter-argument which is $\langle H', \neg f \rangle$.

Given a set of beliefs and a set of desires, let us now consider another application which is decision making and where things look different. The basic idea behind decision making is, to define an ordering between different alternatives, called *decisions*. In [AP04c] the decisions are based on the comparison of arguments and counter-arguments which are defined in different ways. The idea is that a decision is justified if it leads to the satisfaction of some desires, and it is not justified if it violates some desires.

Let $\mathcal{D} = \{d_i; i = 1, ..., m\}$ represent the desires of the decision-maker and \mathcal{D}_e be a set of decisions. Elements of \mathcal{D} and \mathcal{D}_e are formulas of \mathcal{L} .

Definition 2 (Argument for a decision) An argument in favour of a decision is a triple $A = \langle S, C, d \rangle$ such that:

- $d \in \mathcal{D}_e$
- $S \subseteq \mathcal{K}$ and $C \subseteq \mathcal{D}$
- $S \cup \{d\}$ is consistent
- $S \cup \{d\} \vdash C$
- S is minimal and C is maximal (for set inclusion) among the sets S and C satisfying the four previous items.

S = Premises(A) is the premises of the argument and C = Consequences(A) its consequences (the desires which are reached by the decision d).

Arguments against a decision, however, show clearly that desires will not be satisfied by that decision. Formally:

Definition 3 (Argument against a decision) An argument against a decision is a triple $A = \langle S, C, d \rangle$ such that:

- $d \in \mathcal{D}_e$
- $S \subseteq \mathcal{K}$ and $C \subseteq \mathcal{D}$
- $S \cup \{d\}$ is consistent
- $\forall g_i \in C, S \cup \{d\} \vdash \neg g_i$
- S is minimal and C is maximal (for set inclusion) among the sets S and C satisfying the four previous items.

S = Premises(A) is the premises of the argument and C = Consequences(A) its consequences (the desires which are not satisfied by the decision d).

Example 3 The example is about taking an umbrella or not, knowing that the sky is cloudy. The knowledge base is $\mathcal{K} = \{u \to \neg w, r \land \neg u \to w, c, \neg r \to \neg w, c \to r\}$ with: r: it rains, w: being wet, u: taking an umbrella, c: the sky is cloudy.

The base of desires is $\mathcal{D} = \{\neg w, \neg u\}$. There is one argument in favour of the decision "u": $\langle \{u \rightarrow \neg w\}, \{\neg w\}, u \rangle$ and one argument against the decision "u": $\langle \emptyset, \{\neg u\}, u \rangle$.

Note also that the above two arguments are handled in different ways. In [AP04a], it has been shown that the use of arguments in favour of a decision is sufficient to capture the results of the pessimistic criteria defined in [DP95] in qualitative decision making. Whereas, the use of arguments against a decision allows us to capture the results of the optimistic criteria.

3.2 Characterizing bipolarity in the building of arguments

Bipolarity appears in this step of the argumentation process under the form of arguments in favour of a conclusion and arguments against that conclusion. Depending on the application, there exist two possibilities:

Depending on the application, there exist two possibilities:

- either the arguments against and arguments in favour of a conclusion are defined in the same way (see the case of explanatory arguments),
- or there are two distinct ways for defining arguments in favour and arguments against (see the case of arguments for decision).

Here, bipolarity has the following characteristics:

- exclusivity: an argument cannot be at the same time in favour of and against a conclusion³ (it is a trivial property due to the definitions of arguments);
- duality: in the case of explanatory arguments, the duality is obvious: an argument against a proposition is computed from an argument in favour of another proposition; in the case of decision context, a weak duality also exists because all arguments are built using the same data, and, for a given decision d, a partition of the set of all possible arguments exists (the set of arguments in favour of d, the set of arguments against d, the set of arguments neither in favour of d, nor against d); in this case, the duality is a consequence of the exclusivity property: an argument in favour of d cannot be an argument against d (and vice-versa);
- no exhaustivity: arguments that are neither in favour of nor against the conclusion may exist;
- the computation of these two kinds of arguments is made on the same data,
- but not always with the same process.

So, bipolarity in the building of arguments is a type 2 bipolarity in the sense of [DP06] but with a particular property: the exclusivity.

4 Bipolarity at the interaction level

4.1 Building of the interactions between arguments

As already said, due for instance to the presence of inconsistency in knowledge bases, arguments may be conflicting. Indeed, in all argumentation systems, a defeasibility relation is considered in order to capture that conflicts.

However, most logical theories of argumentation assume that: if an argument A_1 defeats an argument A_3 and A_3 defeats an argument A_2 , then A_1 supports A_2 . The notion of support does

 $^{^3 {\}rm This}$ conclusion can be a formula, a decision, . . .

not have to be formalized in a way really different from the notion of defeat⁴. It is a parsimonious strategy, but it is not a correct description of the process of argumentation, because they do not take into account two main properties of argumentation.

- First, argumentation is dependent on relevance in many respects.
- Second, support does not use the same method as attack. Counter-attack is not the same thing as support.

Let us take an example of the first property:

We want to go hiking. We prefer a sunny weather, then a sunny and cloudy one, then a cloudy but not rainy weather, in this order. We will cancel the hiking only if the weather is rainy. But clouds could be a sign of rain. We look at the sky early in the morning. It is cloudy.

One of us says: "These clouds are early patches of mist, the day will be sunny, without clouds". This argument supports the claim: "the weather will be sunny".

Another one says: "clouds will not grow". This is an argument for the claim: "the weather will be cloudy, but not rainy".

Both arguments support the hiking project. But these arguments are not compatible, as one is in favour of a sunny weather without clouds, and the other one in favour of a cloudy weather. The first one is a counter-counter-argument against "the weather will be cloudy, clouds are a sign of rain, we would have better to cancel the hiking". The other is a counter-counter-argument against "the weather will be sunny". In this respect, it could be an argument against the previous argument, so some logical formalism would count it as reinstalling the counter-argument: "clouds are a sign of rain, we would have better to cancel the hiking". But in fact it was used as an argument telling that even if there are clouds, they do not develop, so that the risk of rain is less than this counter-argument about clouds as a sign of rain might let us suppose. A counter-countercounter-argument may reinstall the first claim instead of defeating it. And even if an argument defeats a counter-counter-argument, even if these two arguments are defeating each one another, they can be arguments in support of the same claim.

This first property of argumentation shows that the idea of a chain or arguments and counterarguments in which we just have to count the links and take the even one as defeaters and the odd ones as supports is an oversimplification. So, the notion of defence proposed by [Dun95] and recalled in Section 2 is insufficient.

The second property leads us to the notion of bipolarity:

Suppose that we take information from different weather services and that the first one tells: "cloudy but with sunny spells", the second one: "mainly cloudy", and the third one: "cloudy".

We may take the worst forecast but say: "well, even if the weather is cloudy, so long as it is not rainy the hiking is possible". This is a *defensive argumentation*. We anticipate attacks, we do not give counter-arguments against them, we accept that our position will be weakened, and we save the conclusions that can resist even in such a weakened position. This defensive stance is reminiscent of the defender's attitude in Aristoteles' dialectic. The reason for such a defensive stance is that the claim is supposed to be open to future revisions that we can anticipate.

Suppose now that one of us says :"look, the clouds are clearing, let's go for a walk". Here the counter-argument against the hiking ("it is cloudy, clouds are a sign of rain, no hiking when it is

 $^{^{4}}$ It is the case of the basic argumentation context recalled in Section 2, in which only one kind of interaction is explicitly represented by the *defeat* relation.

In this context, the support of an argument A by another argument B can be represented only if B defends A in the sense of [Dun95]. So, support and defeat are *dependent* notions.

rainy") is defeated by a new fact, not by another argument. This new fact undermines the factual support of the counter-argument ("it is cloudy"). Here what has been revised is not our claim, but the support of its counter-argument. A reasonable strategy seems then to be the following: as long as revisions are anticipated as possible and opportunities do not have yet been given for all these revisions, take the defensive stance.

As soon as these opportunities have been given and the revision turned to undermine the support of the counter-arguments, we take our claims and their support to be sufficiently robust against revisions, contrary to their counter-arguments, take the *positive stance*. This positive stance consists in taking at least some of our claims as robust enough to be the basis of counter-attacks: "as the clouds have gone, the weather is at least better than cloudy, so that the degree of satisfaction of the hiking will be better than its minimum (hiking on a cloudy day)".

Notice that as long as an argument has not been exposed to revision, its possibility is not guaranteed, but that direct perception is in the normal cases (let aside hallucinations and the like) a source of robustness against revision, because it carries with itself the counter-argument against a revision: "this is not direct perception". Of course, even direct perception can be defeated, but only by other arguments robust against revision, that is, by arguments the possibility of which is itself guaranteed.

In a nutshell, the argumentation game always uses arguments and counter-arguments, support and defeat relations, but not always in the same way:

- either predictable sources of defeat are anticipated and positions revised to their minimal claims,
- or revisions have turned out against the support of the counter-arguments and then we can take our claims as robust and try to raise their strength.

The first stance corresponds to taking the *minimal measure of possibility*, and the second one corresponds to accumulate stronger and stronger claims as soon as *their possibility can be guaranteed*. The first method is well adapted for claims that are considered as not sufficiently checked and exposed to revisions, the second one is well adapted for claims the defeaters of which have been themselves revised.

Following all these remarks, and in order to represent realistic examples in an argumentation context, we need a more powerful tool than the abstract argumentation framework proposed by Dung.

In particular, we are interested in modelling situations where two *independent* kinds of interactions are available: a positive and a negative one (see for example in the medical domain the work [KP01]). So, following [KP01, Ver02], we propose a new argumentation framework: an abstract bipolar argumentation framework.

4.1.1 An abstract bipolar argumentation framework

An abstract bipolar argumentation framework is an extension of the basic argumentation framework introduced by [Dun95] in which we use a new kind of interactions between arguments: the *support* relation which represents the support, the help brought by some arguments to other arguments⁵. This new relation is totally independent of the defeat relation. So, we have a bipolar representation of the interactions between arguments.

Formal definition of an abstract bipolar argumentation framework An abstract bipolar argumentation framework $\langle \mathcal{A}, \mathcal{R}_{def}, \mathcal{R}_{sup} \rangle$ consists of a set \mathcal{A} of arguments, a binary relation \mathcal{R}_{def} on \mathcal{A} called a *defeat relation* and another binary relation \mathcal{R}_{sup} on \mathcal{A} called a *support relation*: consider A_i and $A_j \in \mathcal{A}, A_i \mathcal{R}_{def} A_j$ (resp. $A_i \mathcal{R}_{sup} A_j$) means that A_i defeats A_j (resp. A_i supports A_j).

 $^{^5\}mathrm{If}$ the support relation is removed, we retrieve Dung's framework.

A bipolar argumentation framework is called *well-founded* if and only if there is no infinite sequence $A_0, A_1, \ldots, A_n, \ldots$ such that $\forall i, A_i \in \mathcal{A}$ and $A_{i+1}\mathcal{R}A_i$ with $\mathcal{R} = \mathcal{R}_{def}$ or \mathcal{R}_{sup} .

Here, we are not interested in the structure of the arguments and we consider arbitrary defeat and support relations. The only assumption is that \mathcal{R}_{def} and \mathcal{R}_{sup} are independent of each other.

Notations Consider $A \in \mathcal{A}$, $A\mathcal{R}_{def}B$ is represented by $A \not\rightarrow B$ and $A\mathcal{R}_{sup}B$ is represented by $A \rightarrow B$. The set $\{A_i \in \mathcal{A} | A_i \mathcal{R}_{def}A\}$ is denoted by $\mathcal{R}_{def}^-(A)$ and the set $\{A_i \in \mathcal{A} | A\mathcal{R}_{def}A_i\}$ is denoted by $\mathcal{R}_{def}^+(A)$. In the same way, we define $\mathcal{R}_{sup}^-(A)$ and $\mathcal{R}_{sup}^+(A)$. $\langle \mathcal{A}, \mathcal{R}_{def}, \mathcal{R}_{sup} \rangle$ defines a directed graph \mathcal{G}_b called the *bipolar graph*.

Example 4 The framework $\langle \mathcal{A} = \{A_1, A_2, A_3, A_4\}, \mathcal{R}_{def} = \{(A_2, A_3), (A_4, A_3), (A_1, A_2)\}, \mathcal{R}_{sup} = \{(A_2, A_4), (A_1, A_3)\} > defines the following graph <math>\mathcal{G}_b$ with the root A_3 :



Definition 4 (Graphical representation of a bipolar argumentation framework) Let \mathcal{G}_b be the bipolar graph associated with the abstract bipolar argumentation framework $\langle \mathcal{A}, \mathcal{R}_{def}, \mathcal{R}_{sup} \rangle$, we define:

- Leaf of the bipolar graph A leaf of \mathcal{G}_b is an argument $A \in \mathcal{A}$ such that $\mathcal{R}_{def}^-(A) = \emptyset$ and $\mathcal{R}_{sup}^-(A) = \emptyset$.
- Path in the bipolar graph A path from A to B is a sequence of arguments $\mathcal{P} = A_1 \dots A_n$ such that $A = A_1, A_1 \mathcal{R}_1 A_2, \dots, A_{n-1} \mathcal{R}_{n-1} A_n, A_n = B$, and $\forall i = 1, \dots, n-1$, $\mathcal{R}_i = \mathcal{R}_{def}$ or \mathcal{R}_{sup} .

The set of the paths from A to B will be denoted by $\mathcal{P}(A, B)$.

- Length of a path The length of the path $\mathcal{P} = A_1 \ldots A_n$ is n-1 (the number of edges that are used in the path) and will be denoted by $l_{\mathcal{P}}$.
- Defeat and support numbers of a path The defeat number of the path (resp. support number of the path) $\mathcal{P} = A_1 \ldots A_n$, with $A_1 \mathcal{R}_1 A_2, \ldots, A_{n-1} \mathcal{R}_{n-1} A_n$, is the number of $\mathcal{R}_i = \mathcal{R}_{def}$ (resp. $\mathcal{R}_i = \mathcal{R}_{sup}$) and will be denoted by $n_{def}(\mathcal{P})$ (resp. $n_{sup}(\mathcal{P})$).
- Homogeneous path A homogeneous path from A to B is a path in which all the \mathcal{R}_i are the same. So, we can have homogeneous defeat paths or homogeneous support paths.
- Branch for an argument A path from A to B is a branch for B iff A is a leaf of \mathcal{G}_b .

We propose the notions of direct and indirect defeaters and defenders⁶, completed with the notion of direct and indirect supporters. Note that negative information (defeat edges) is considered as having priority over positive information (support edges). So, we do not have symmetrical definitions for indirect defeaters/defenders and indirect supporters⁷:

Definition 5 (Direct/Indirect Defeaters/Defenders of an argument) Consider $A \in \mathcal{A}$:

- The direct defeaters of A are the elements of $\mathcal{R}_{def}^{-}(A)$.
- The direct defenders of A are the direct defeaters of the elements of $\mathcal{R}_{def}^{-}(A)$.
- The indirect defeaters of A are the elements A_i defined by:

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\exists \mathcal{P} \in \mathcal{P}(A_i, A) \text{ such that } n_{def}(\mathcal{P}) = 2k + 1,
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with $k \geq 0$ and A_i is not a direct defeater.

 $^{^{6}}$ The notions introduced here are inspired by related definitions first introduced in [Dun95] but are not strictly equivalent: in [Dun95]'s work, direct defeaters (resp. defenders) are also indirect defeaters (resp. defenders) which is not true in our definitions.

⁷As soon as the path $A_i - A$ contains at least one defeat edge, it defines A_i as an indirect defeater or defender for A. Contrastedly, an indirect supporter A_i for A excludes defeat edges in the path $A_i - A$. This illustrates that bipolarity is well represented by the defeat and support relations while no bipolarity exists with the defeat and defend relations.

• The indirect defenders of A are the elements A_i defined by: $\exists \mathcal{P} \in \mathcal{P}(A_i, A) \text{ such that } n_{def}(\mathcal{P}) = 2k,$ with $k \geq 1$ and A_i is not a direct defender.

Definition 6 (Direct/Indirect supporters of an argument) Consider $A \in A$:

- The direct supporters of A are the elements of $\mathcal{R}_{sup}^{-}(A)$.
- The indirect supporters of A are the elements A_i defined by: $\exists \mathcal{P} \in \mathcal{P}(A_i, A) \text{ such that } n_{sup}(\mathcal{P}) = l_{\mathcal{P}} \geq 2.$

All these notions are illustrated on the following example:

Example 5

A'

D0 D1

E2

On this graph \mathcal{G}_b which is not connected, we see:



- the paths $D_1 C_1 B_1$ are $C_3 B_2 A$ are independent, the paths $D_1 C_1 B_1 A$ and $C_3 B_2 A$ are root-dependent and the paths $D_1 C_1 B_1 A$ and $C_2 B_1 A$ are dependent,
- the path $E_1 D_2 C_3 B_2 A$ has $n_{def} = 4$ and $n_{sup} = 0$,
- the path $E_1 D_4 A_6 A_5 A$ has $n_{def} = 1$ and $n_{sup} = 3$,
- D_1 , E_2 , E_1 and E_0 are the leaves of \mathcal{G}_b ,
- B_1 and B_2 are the only direct defeaters of A,
- C_1 , C_2 and C_3 are the only direct defenders of A,
- E_0 , D_0 , D_1 , D_2 , D_4 , E_1 and E_2 are the only indirect defeaters of A,
- E_1 and D_3 are the only indirect defenders of A,
- A_5 is the only direct supporter of A,
- A_6 is the only indirect supporter of A,
- A' is not related to A.

4.1.2 Remarks and examples

E1

D3

E0

In a bipolar argumentation framework, the support relation carries positive information while the defeat relation carries negative information, and positive and negative information are represented in the same structure (the bipolar graph). It is a particularity of the argumentation context (in many other domains, positive and negative information are represented in two distinct frameworks, and sometimes they do not have the same nature).

There exist many different formal definitions for these relations. Using the explanatory arguments proposed in Definition 1, Section 3, we give here the most useful definitions:

Definition 7 (Defeat relations) Let A_1 and A_2 be two explanatory arguments ($A_1 = \langle H_1, h_1 \rangle$ and $A_2 = \langle H_2, h_2 \rangle$).

We have at least two kinds of defeat relations:

- $A_1 \mathcal{R}_{def} A_2$ iff $\exists \phi \in H_2$ such that $\phi \equiv \neg h_1$ (undercut relation).
- $A_1 \mathcal{R}_{def} A_2$ iff $\neg h_2 \equiv h_1$ (rebut relation).

Definition 8 (Support relations) Let A_1 and A_2 be two explanatory arguments ($A_1 = \langle H_1, h_1 \rangle$ and $A_2 = \langle H_2, h_2 \rangle$).

We have at least two kinds of support relations:

• $A_1 \mathcal{R}_{sup} A_2$ iff $\exists \phi \in H_2$ such that $\phi \equiv h_1$ and $H_1 \cup H_2$ is consistent (explanation support).

• $A_1 \mathcal{R}_{sup} A_2$ iff $h_2 \equiv h_1$ and $H_1 \cup H_2$ is consistent (conclusion support).

We present below some illustrative examples.

Example 6 (this example was introduced for the first time in [Amg99, AM00]) During a discussion between reporters about the publication of an information I concerning the person X, the following arguments are presented:

- A: I is an important information, we must publish it.
- B: I concerns the person X, X is a private person and we can not publish an information about a private person without her agreement, and X does not agree with the publication.
- C: X is a minister, so X is a public person, not a private person.
- D: X has resigned, so X is no more a minister.
- E: her resignation has been refused by the chief of the government.
- F: I concerns a problem of public health, so I is an important information.



In this example, B is a direct defeater of A, F is a direct supporter of A, C is a direct defender of A, D is an indirect defeater of A and E is an indirect defender of A.

Example 7 During a discussion between doctors about the installation of a prosthesis on the patient X, the following arguments are presented:

- A: X has difficulties for walking, we must install a prosthesis.
- B: the installation of a prosthesis needs a surgical operation with an anaesthesia which is very risked for the patient and we do not want to take a risk.
- C: we can use a local anaesthesia, so there is no more risk.
- D: a surgical operation presents also important risks of post-infections.
- E: there are more and more kinds of nosocomial infections in the hospital and it is very difficult to cure them.
- *F*: the classical treatments (injections) are unable to cure X's knee problem, we must install a prosthesis.



In this example, B and D are direct defeaters of A, F is a direct supporter of A, C is a direct defender of A and E is an indirect defeater of A.

Example 8 The following discussion between 3 agents (Tom, Ben and Dan) about a hiking is presented:

- T₁: Today we have time, we go hiking.
- B: No, the weather is cloudy, clouds are a sign of rain, it is more cautious to cancel the hiking.
- T₂: These clouds are early patches of mist, the day will be sunny without cloud, so we can begin the hiking.
- D: No, these clouds are not early patches of mist. So, the day will not be sunny but cloudy. However, it will not rain, so we can begin the hiking.



4.2 Characterizing bipolarity in the interactions between arguments

In this step, bipolarity appears through the introduction of two kinds of interaction on the set of arguments. These relations are independently built and they represent:

- a conflict relation between the arguments,
- a support relation between the arguments.

Using these relations, we can extend the abstract argumentation system proposed by [Dun95] in an abstract bipolar argumentation system.

So, we have two subgraphs (one for each relation) which can be represented on the same graphic structure.

Bipolarity has the following characteristics:

- exclusivity: a consistency constraint forbids an argument which simultaneously supports and defeats another argument⁸;
- duality: the support relation (resp. defeat relation) is not inferred from the defeat relation (resp. support relation); however a weak duality exists because, apart from the supporting arguments, one could consider arguments which are not conflicting with a given argument;
- no exhaustivity: arguments that do not interact may exist;
- the computation of these two kinds of interactions is not always made on the same data,
- and not always with the same process.

So, bipolarity at the interaction level is at least a type 3 bipolarity but also, sometimes a type 2 bipolarity (when the computation of conflict relation and support relation is made on the same data). In all cases, a particular property of exclusivity exists due to a consistency constraint between positive and negative information.

 $^{^{8}}$ In argumentation framework, this consistency constraint is essential: in order to simulate a safe reasoning made by rational agents, we cannot use an argument which simultaneously defeats and supports another argument.

5 Valuation in a bipolar argumentation framework

Bipolarity reflects the nature of the interactions (positive interaction, negative interaction and of course absence of interaction). Then the argumentation process requires a valuation mechanism in order to select the most acceptable arguments.

Within Dung's framework, several approaches have been proposed for valuating the arguments, using or not the only defeat relation⁹, see for example [KAEF95, Par97, PS97, JV99, BH01, CLS03b, Amg99].

In the context of a bipolar argumentation framework as defined in Section 4, the valuation follows the same principles as those that have already been described in [CLS03b] completed with new principles corresponding to the "support" information. Here, we propose two kinds of valuations:

- a *local valuation* in which the value of an argument only depends on the values of the direct defeaters or supporters of this argument.
- a *global valuation* in which all the branches leading to this argument are considered when computing the value of the argument.

In order to simplify the valuation process, we assume that the bipolar graph is acyclic.

5.1 Local valuation in a bipolar argumentation framework

In the local approach, we follow some principles:

- **Pl1** The valuation of an argument depends on the values of its direct defeaters and of its direct supporters.
- **Pl2** If the quality¹⁰ of the support (resp. defeat) increases then the value of the argument increases (resp. decreases).
- **Pl3** If the quantity¹¹ of the supports (resp. defeats) increases then the quality of the support (resp. defeat) increases.

While respecting these principles, we assume that there exists a completely ordered set V with a minimum element (V_{Min}) and a maximum element (V_{Max}) and we propose the following formal definition for a local gradual valuation.

Definition 9 Let $\langle \mathcal{A}, \mathcal{R}_{def}, \mathcal{R}_{sup} \rangle$ be a bipolar argumentation framework. Let $A \in \mathcal{A}$ with $\mathcal{R}_{def}^{-}(A) = \{B_1, \ldots, B_n\}$ and $\mathcal{R}_{sup}^{-}(A) = \{C_1, \ldots, C_p\}$.

A local gradual valuation on $\langle \mathcal{A}, \mathcal{R}_{def}, \mathcal{R}_{sup} \rangle$ is a function $v : \mathcal{A} \to V$ such that

$$v(A) = g(h_{sup}(v(C_1), \dots, v(C_p)), h_{def}(v(B_1), \dots, v(B_n)))$$

with:

the function h_{def} (resp. h_{sup}): $V^* \to \mathcal{H}_{def}$ (resp. $V^* \to \mathcal{H}_{sup}$)¹² valuating the quality of the defeat (resp. support) on A.

the function $g : \mathcal{H}_{sup} \times \mathcal{H}_{def} \to V$ with g(x, y) increasing on x and decreasing on y.

The function $h, h = h_{def}$ or h_{sup} , must respect the following constraints:

- if $x_i \ge x'_i$ then $h(x_1, \ldots, x_i, \ldots, x_n) \ge h(x_1, \ldots, x'_i, \ldots, x_n)$,
- $h(x_1, ..., x_n, x_{n+1}) \ge h(x_1, ..., x_n),$

⁹So, the value of an argument may depend on its interactions with the other arguments, or may depend on an intrinsic strength of the argument which can be defined for example by an explicit preference.

 $^{^{10}{\}rm The}$ quality represents the global strength of the supporting (resp. defeating) arguments.

 $^{^{11}{\}rm The}$ quantity represents the number of the supporting (resp. defeating) arguments.

 $^{^{12}}V^*$ denotes the set of the finite sequences of elements of V. \mathcal{H}_{def} and \mathcal{H}_{sup} are ordered sets.

- $h() = \alpha \le h(x_1, \dots, x_n)$, for all x_1, \dots, x_n^{13} ,
- $h(x_1, ..., x_n) \le \beta$, for all $x_1, ..., x_n^{-14}$.

Definition 9 enables positive and negative information about an argument to be combined into a single value in order to compare efficiently different arguments. This definition takes into account the arguments which are directly related to the argument under consideration.

Note that Definition 9 produces a generic local gradual valuation. There exist several instances for this generic valuation:

- One of them is the following:
 - $\mathcal{H}_{def} = \mathcal{H}_{sup} = V = [-1, 1]$ interval of reals,
 - $h_{def}(x_1, ..., x_n) = h_{sup}(x_1, ..., x_n) = \max,$
 - $g(x,y) = \frac{x-y}{2}$.

So, we have $\alpha = -1$, $\beta = 1$ and $g(\alpha, \alpha) = 0$.

- Another one is the following:
 - V = [-1, 1] interval of reals,
 - $\mathcal{H}_{def} = \mathcal{H}_{sup} = [0, \infty]$ interval of reals,
 - $h_{def}(x_1, \ldots, x_n) = h_{sup}(x_1, \ldots, x_n) = \sum_{i=1}^n \frac{x_i + 1}{2},$
 - $g(x,y) = \frac{1}{1+y} \frac{1}{1+x}$.

So, we have $\alpha = 0$, $\beta = \infty$ and $g(\alpha, \alpha) = 0^{15}$.

Examples

In Example 6, Section 4.1.2, we obtain:

- with the first instance, $v(A) = \frac{3}{16}$,
- and with the second instance, $v(A) = \frac{1}{15}$.

In Example 7, Section 4.1.2, we obtain:

- with the first instance, $v(A) = \frac{-1}{4}$,
- and with the second instance, $v(A) = \frac{-1}{6}$.

In Example 8, Section 4.1.2, we obtain:

- with the first instance, $v(T_1) = \frac{1}{4}$,
- and with the second instance, $v(T_1) = \frac{37}{154}$.

5.2 Global valuation in a bipolar argumentation framework

As said before, the basic idea behind a global valuation is to take into account the whole graph of interactions when valuating an argument. The valuation function should agree with the following principles:

Pg1 The value of a given argument is returned by a function which takes into account all the branches leading to this argument in the bipolar graph.

 $^{^{13}}$ So, α is the minimal value for a defeat (resp. a support) – *i.e.* there is no defeat (resp. no support) –.

¹⁴So, β is the maximal value for a defeat (resp. a support) – *i.e.* for example, if there is an infinity of direct defeaters (resp. supporters) –.

¹⁵Note that $h_{def}(x_1, \ldots, x_n, x_{n+1}) \ge h_{def}(x_1, \ldots, x_n)$ because $\frac{x_{n+1}+1}{2} \ge 0$ when $x_{n+1} \in [-1, 1]$ (and the same for h_{sup}). We have also $h_{def}() = h_{sup}() = \alpha$, α being the minimal value of $[0, \infty[$, and β being the maximal value of $[0, \infty[$. We can verify also that $g(\alpha, \beta) = g(0, \infty) = -1$ and that $g(\beta, \alpha) = g(\infty, 0) = 1$ (1 and -1 being respectively V_{Min} and V_{Max}).

- **Pg2** The set of the branches leading to this argument is partitioned into three parts. Each part corresponds to a kind of "effect" on the argument. Three "effects" can be distinguished: the "defeat part", the "defence part" and the "support part".
- **Pg3** The improvement of the defence or the support parts or the degradation of the defeat part of an argument leads to an increase of the value of this argument.
- **Pg4** The improvement of the defeat part or the degradation of the defence or the support parts of an argument leads to a decrease of the value of the argument.

Note that the defence and the support parts have a positive flavour whereas the defeat part has a negative one.

The main problem is how to determine the "effect" of a branch leading to an argument ? There exist different methods and each of them leads to a kind of global valuation. Here, we will present only one of these methods.

Definition 10 (Defeat, defence and support branches for an argument) Let A be an argument.

- **Defeat (resp. defence) branch:** A branch for A is a defeat branch (resp. defence branch) iff the longest homogeneous path leading to A and containing A of this branch is an homogeneous defeat path whose length is an odd integer (resp. even integer).
- **Support branch:** A branch for A is a support branch iff the longest homogeneous path leading to A and containing A of this branch is an homogeneous support path.
- **Useful length of a branch:** It is the length of the longest homogeneous path which determines the nature of the branch.

Examples:

In Example 6, Section 4.1.2, E - D - C - B - A is a defence branch for A (its useful length is 4) and F - A is a support branch for A (its useful length is 1).

In Example 7, Section 4.1.2, C - B - A is a defence branch for A (its useful length is 2), E - D - A is a defeat branch for A (its useful length is 1) and F - A is a support branch for A (its useful length is 1).

In Example 8, Section 4.1.2, $D - B - T_1$ is a defence branch for T_1 (its useful length is 2), $D - T_2 - B - T_1$ is a defeat branch for T_1 (its useful length is 3) and $D - T_1$ and $D - T_2 - T_1$ are support branches (their useful lengths are 1).

Using the above definition, we can define the value of an argument under the form of three values (one for each part evoked in the principles – defeat part, defence part, support part of the set of the branches leading to the argument). These values must represent all the branches leading to the argument). These values must represent all the branches leading to the argument and having the same effect. Following the encoding used in [CLS03b] in the case of a basic argumentation framework, we can consider each value as a tuple of pieces of information (one piece of information for each branch). In [CLS03b], a tuple only recorded the lengths of branches, but here, it is more complex because of the mix of support edges and defeat edges in a same branch. So, one idea is to encode the description of the branch under the form of a sequence of bits (1 for a defeat edge and 0 for a support edge). With this encoding, it is very easy to distinguish between defeat/defence branches and support branches.

Example:

In Example 8, the value of T_1 will be:

$$(\underbrace{[0,10]}_{\text{support-value defeat-value defence-value}}, \underbrace{[111]}_{\text{defence-value defence-value}})$$

Complete definitions of these three tupled-values and some comparison algorithms on these values respecting the proposed principles can be found in [CLS04]. The main idea of these comparison algorithms is that the *comparison of two arguments* proceeds in two steps¹⁶:

- The "first step" compares the number of defeat branches, the number of support branches and the number of defence branches of each argument. Following the proposed principles, we have two criteria: one for the defence and the support (positive criterion) and another for the defeat (negative criterion). These criteria are aggregated using a *cautious approach*, *i.e.* we conclude if one of the arguments has more defence and support branches (it is better according to the positive criterion) and less defeat branches than the other argument (it is also better according to the negative criterion). Note that we conclude positively only when all the criteria agree: if one of the arguments has more defence and support branches (it is better according to the positive criterion) and more defence than the other argument (it is worse according to the negative criterion), the arguments are considered to be incomparable.
- Else, the arguments have the same number of defence branches, the same number of support branches and the same number of defeat branches, and a "second step" *compares the quality of the defeats, the quality of the supports and the quality of the defences* using the useful length of each branch. This comparison is made lexicographically and gives two criteria which are again aggregated using a cautious method. In case of disagreement, the arguments are considered to be incomparable.

Example:

v(A) in Example 6 is better than v(A) in Example 7: in Example 7, A has one support branch, one defence branch and one defeat branch, and in Example 6, A has one support branch, one defence branch and does not have a defeat branch.

5.3 Characterizing bipolarity in the valuation of the arguments

In this step, we use bipolar data (the abstract bipolar argumentation system defined in Section 4) and we propose two kinds of bipolar valuation:

• the local valuation enables two different kinds of information, support and defeat, to be processed separately through the functions h_{def} and h_{sup} . Each function pertains to a separate unipolar scale. So we can distinguish between "ignorance" ('no positive information and no negative information) and "indifference" (as much information positive as negative information). Then, in order to compare the arguments more efficiently, we recover a bipolar univariate scale with the function g (positive and negative information are combined). It corresponds to the following figure:



This mechanism is convenient in practice. However, when using g, it is no longer possible to distinguish between ignorance and indifference, which may appear as a severe drawback, even if counterbalanced by the efficiency of the comparison.

Bipolarity with a local approach for valuation has the following features:

- exclusivity: the value of an argument cannot be at the same time positive and negative;
- duality appears only in the second step: in the first step of the local valuation, no duality exists (the positive resp negative value of an argument is not deduced from the negative resp. positive value of another argument¹⁷); nevertheless, in the

¹⁶It is the extension of the algorithm presented in [CLS03b] when there is only one defeat relation and no support relation.

¹⁷Even if we can identify some symmetrical situations in which a kind of duality exists – see [CLS04].

second step, after a drastic simplification, a strong duality appears (the negative feature becomes a reflection of the positive one);

- exhaustivity: the value of an argument is either positive or negative (exclusive or);
- the computation of these two kinds of values is made on the same data (the interaction graph) but not always with the same process (even if in the presented examples we use exactly the same function).

At the local valuation level, bipolarity first appears as a type 3 bipolarity, and then, after a drastic simplification, as a type 1 bipolarity.

• the global valuation in which, although there are three tupled-values per each argument, we have two unipolar bivariate scales, because the defence-value and the support-value are used together "against" the defeat-value.



Note also that, with the global approach, we can distinguish the case "balance between defeats and supports-defence" and the case "no defeat and no support-defence". Bipolarity with a global approach for valuation has the following features:

- no exclusivity: the value of an argument can be at the same time positive¹⁸ and negative¹⁹;
- no duality: the positive (resp. negative) value of an argument is not deduced from the negative (resp. positive) value of another argument;
- exhaustivity: the value of an argument is either positive or negative (no exclusive or);
- the computation of these two kinds of values is made on the same data (the interaction graph), but not always with the same process.

Within a global valuation, bipolarity appears as a type 3 bipolarity with two unipolar bivariate scales (as a type 2 bipolarity).

6 Bipolarity in the selection of the acceptable arguments

Bipolarity also appears when defining the acceptability of arguments, even in the case of a basic argumentation framework and, of course, also in the case of a bipolar argumentation framework. This step consists in identifying a partition of the set of the arguments. This partition can be more or less rich, but generally an argumentation process should return three categories of arguments:

- The class of *acceptable* arguments. Beliefs or goals or decisions supported by such arguments are really justified. In the case of inconsistency handling in knowledge bases, beliefs supported by such arguments will be inferred from the base. Similarly, goals supported by such arguments will be pursued by the agent.
- The class of *rejected* arguments. For example, goals supported only by such arguments will be rejected by the agent even if they can be achieved. Decisions supported by such arguments will be discarded and beliefs supported by such arguments will not be inferred from the knowledge base.
- The class of arguments in abeyance. Such arguments are neither acceptable nor rejected.

 $^{^{18}}i.e.$ there is a support-defence part in the value.

 $^{^{19} \}it{i.e.}$ there is a defeat part in the value.

Depending on the nature of the arguments and the application which is considered, different kinds of bipolarity can be distinguished when defining the acceptability of arguments. First, we present the case of a basic argumentation framework with two distinct examples. Then, we show how to define acceptability in the case of a bipolar argumentation framework.

6.1 Case of a basic argumentation framework

In this context, there are two kinds of bipolarity when defining the acceptability of arguments. In the first one, the two classes of rejected arguments and arguments in abeyance are defined on the basis of the class of acceptable arguments. For example, in the case of inconsistency handling in knowledge bases, the grounded extension (introduced in Section 2) may be used to define the acceptable arguments.

Definition 11 An argument is acceptable if it belongs to the grounded extension and an argument is rejected if it is attacked by an acceptable argument.

In this case, rejected arguments are defined using acceptable arguments and there do not exist arguments in abeyance.

In the second kind of bipolarity, the classes of acceptable and rejected arguments are defined separately and the class of arguments in abeyance is deduced from these two classes. We illustrate this kind of bipolarity in a particular application where we try to compute the intentions of an agent from its contradictory desires (see [Amg03] for more details).

Let \mathcal{L} be a propositional language, an agent is supposed to be equipped with a base \mathcal{D} of desires, a belief base Σ containing the plans to carry out in order to achieve the desires (we are not interested in the way in which these plans are generated), and finally a base \mathcal{C} of integrity constraints.

- \mathcal{D} contains literals of \mathcal{L} . The elements of \mathcal{D} represent the initial desires of the agent. For example, an agent may have the following desires: to finish a publication, to go to a dentist, etc... Note that the set \mathcal{D} may be inconsistent. This means that an agent is allowed to have contradictory desires.
- Σ contains rules having the form $\varphi_1 \wedge \ldots \wedge \varphi_n \to h$ where $\varphi_1, \ldots, \varphi_n, h$ are literals of \mathcal{L} . Such a formula means that the agent believes that if he realizes $\varphi_1, \ldots, \varphi_n$ then he will be able to achieve h.
- C contains formulas of \mathcal{L} . They represent a kind of integrity constraints.

A desire is any element of \mathcal{D} . A desire *h* may have sub-desires. For example, the desire of "going on a journey to central Africa" may have two sub-desires which are: "getting the tickets" and "being vaccinated". The sub-desire "getting the tickets" may have itself the two following sub-desires: "having a friend who may bring the tickets" and "contacting a travel agency".

Definition 12 (Desire/Sub-desire) Let us consider an agent equipped with the bases < D, Σ , C > .

- 1. \mathcal{D} is the set of the desires of the agent.
- 2. Sub \mathcal{D} is the set of the sub-desires of the agent: A literal $h' \in Sub\mathcal{D}$ iff there exists a rule $\varphi_1 \wedge h' \ldots \wedge \varphi_n \to h \in \Sigma$ with $h \in \mathcal{D}$ or $h \in Sub\mathcal{D}$. In that case, h' is a sub-desire of h.

As noted above, an agent may have one or several ways to achieve a given desire. We bring the two notions together in a new notion of *partial plan*.

Definition 13 (Partial plan) A partial plan for h is a pair $a = \langle h, H \rangle$ such that:

- h is a desire or a sub-desire.
- $H = \{\varphi_1, \ldots, \varphi_n\}$ if there exists a rule $\varphi_1 \wedge \ldots \wedge \varphi_n \to h \in \Sigma$, $H = \emptyset$ otherwise.

The function Desire(a) = h returns the desire or sub-desire of a partial plan a and the function Support(a) = H returns the support²⁰ of the partial plan. \aleph will gather all the partial plans that can be built from $<\mathcal{D}, \Sigma, \mathcal{C}>$.

Note 1 A desire may have several partial plans.

Note 2 Let $a = \langle h, H \rangle$ be a partial plan. Each element of the support H is a sub-desire of h.

Definition 14 A partial plan $a = \langle h, H \rangle$ is elementary iff $H = \emptyset$.

A partial plan shows the actions that should be performed in order to achieve the corresponding desire (or sub-desire). However, the elements of the support of a given partial plan are considered as sub-desires that must be achieved in turn by another partial plan. The whole way to achieve a given desire is called a *complete plan*. A *complete plan* for a given desire d is an AND tree. Its nodes are partial plans and its arcs represent the sub-desire relationship. The root of the tree is a partial plan for the desire d. It is an AND tree because all the sub-desires of d must be considered. When for the same desire, there are several partial plans to carry it out, only one is considered in a tree. Formally:

Definition 15 (Complete plan) A complete plan g for a desire h is a finite tree such that:

- $h \in \mathcal{D}$ and the root of the tree is a partial plan $\langle h, H \rangle$.
- A node $\langle h', \{\varphi_1, \ldots, \varphi_n\} \rangle$ has exactly n children $\langle \varphi_1, H'_1 \rangle, \ldots, \langle \varphi_n, H'_n \rangle$ where $\langle \varphi_i, H'_i \rangle$ is a partial plan for φ_i .
- The leaves of the tree are elementary partial plans.

The function Root(g) = h returns the desire of the root. The function Root(g) returns the set of all the partial plans of the tree g. \mathcal{G} denotes the set of all the complete plans that can be built from the triple $\langle \mathcal{D}, \Sigma, \mathcal{C} \rangle$. The function Leaves(g) returns the set of the leaves of the tree g.

In [Amg03], it has been shown that partial plans may be conflicting for several reasons. These different kinds of conflicts are brought together in a unique relation of *conflict* defined as follows:

Definition 16 (Conflict) Let a_1 and a_2 be two partial plans of \aleph . a_1 conflicts with a_2 iff: $\{Desire(a_1), Desire(a_2)\} \cup Support(a_1) \cup Support(a_2) \cup C \cup \Sigma \vdash \bot.$

More generally, a set of partial plans may be conflicting.

Definition 17 Let $S \subseteq \aleph$. S is conflicting iff $\bigcup_{a \in S} (\{Desire(a)\} \cup Support(a)) \cup C \cup \Sigma \vdash \bot$.

Since partial plans may be conflicting, two complete plans may be conflicting too.

Definition 18 (Attack) Let $g_1, g_2 \in \mathcal{G}$. g_1 attacks g_2 iff $\exists a_1 \in Nodes(g_1)$ and $\exists a_2 \in Nodes(g_2)$ such that a_1 conflicts with a_2 .

More generally we are interested in sets of complete plans such that there is no conflict between their nodes. Formally:

Definition 19 (Conflict-free) Let $S \subseteq \mathcal{G}$. S is conflict-free²¹ iff: $\begin{bmatrix} \bigcup_{g \in S} \ [\bigcup_{a \in Nodes(g)} (Support(a) \cup \{Desire(a)\})] \\ \cup C \cup \Sigma \not\vdash \bot]. \end{bmatrix}$ If $S = \{g\}$, then we say that the complete plan g is conflict-free.

Obviously a desire which has no conflict-free complete plan will be called *unachievable*. This means it is impossible to carry out such a desire.

 $^{^{20}}$ Note that this notion of support is independent of the support relation studied in Sections 4 and 5. It is more related to the notion of premises introduced in Section 3.

 $^{^{21}}$ Note that this notion is not the same one which is defined by [Dun95] and recalled in the first section.

Definition 20 (Unachievable desire) A desire d is unachievable if $\nexists g \in \mathcal{G}$ s.t Root(g) = d and g is conflict-free.

From the preceding definitions, we can now present the formal system for handling conflicting desires of an agent.

Definition 21 (System for handling desires) Let us consider a triple $\langle \mathcal{D}, \Sigma, \mathcal{C} \rangle$. The pair $\langle \mathcal{G}, Attack \rangle$ will be called a system for handling desires (SHD).

A SHD has the same features as an argumentation framework [AC02b]. Inspired by previous work on argumentation theory, we will define acceptable sets of complete plans. Then we will be able to partition the set \mathcal{G} into three categories thus meeting again the notion of bipolarity in the selection step:

- 1. The acceptable set(s) of complete plans. They contain the *good plans* to achieve their corresponding desires. These desires will become the intentions of the agent.
- 2. The class of rejected complete plans. These are the self-attacked ones.
- 3. The class of *complete plans in abeyance* which gathers the complete plans which are neither good nor rejected.

We give below the semantics of "acceptable sets of complete plans".

Definition 22 Let $\langle \mathcal{G}, Attack \rangle$ be a SHD and $S \subseteq \mathcal{G}$. S is an acceptable set of complete plans iff: S is conflict-free and S is maximal (for set inclusion).

6.2 Case of a bipolar argumentation framework

Using the bipolar argumentation framework defined in Section 4, new acceptability semantics will appear. So, we briefly present new methods for computing acceptable arguments.

6.2.1 Defeat, support and conflict

Let $\langle \mathcal{A}, \mathcal{R}_{def}, \mathcal{R}_{sup} \rangle$ be a bipolar argumentation system.

Definition 23 (Supported defeat) A supported defeat is a path $A_1 - \ldots - A_n$, $n \ge 3$, such that $\forall i = 1 \ldots n - 2$, $\mathcal{R}_i = \mathcal{R}_{sup}$ and $\mathcal{R}_{n-1} = \mathcal{R}_{def}$. So, it is a path which contains only one defeat edge $(A_{n-1} - A_n)$ and support edges.

By extension, a homogeneous defeat path whose length is 1 will be also called a supported defeat.

Definition 24 (Defeat/support by a set of arguments in a bipolar AF) Let $S \subseteq A$, let $A \in A$.

- S defeats A iff $\exists B \in S$ such that :
 - either $B\mathcal{R}_{def}A$,
 - or one of the paths from B to A is a supported defeat.
- S supports A iff $\exists B \in S$ such that one of the paths from B to A is a homogeneous support path.

For example, the set $\{C\}$ defeats G, F and supports D, E, and the set $\{A\}$ defeats B, but neither G, nor F:



In the context of a bipolar argumentation system, the notion of $conflict^{22}$ corresponds to an extension of the conflict-free notion proposed by [Dun95]. So, we have:

Definition 25 (Conflict-free set in a bipolar AF) Let $S \subseteq A$. S is conflict-free iff $\not\exists A, B \in S$ such that $\{A\}$ defeats²³ B.

An example :



In this example, the following sets are conflict-free:

- $S_1 = \{A, B, C, F\},\$
- $S_2 = \{A, B, E, F\},\$
- $S_3 = \{D, E, F\}.$

6.2.2 Extensions

Using the previous notions and extending the propositions of [Dun95], we define different acceptability semantics:

Definition 26 (Stable extension in a bipolar AF) Let $S \subseteq A$. S is a stable extension iff S is conflict-free and $\forall A \notin S$, S defeats A.

In the previous example, S_2 is a stable extension.

Definition 27 (Defence by a set of arguments in a bipolar AF) Let $S \subseteq A$. Let $A \in A$. S defends A iff $\forall B \in A$, if $B\mathcal{R}_{def}A$ then $\exists C \in S$ such that $C\mathcal{R}_{def}B$.

Definition 28 (Admissible set in a bipolar AF) Let $S \subseteq A$. S is admissible iff S is conflictfree, closed for \mathcal{R}_{sup} and defends all its elements.

In the previous example, S_1 , S_2 are not admissible and S_3 is admissible.

Definition 29 (Preferred extension in a bipolar AF) A set $E \subseteq A$ is a preferred extension iff E is inclusion-maximal among the admissible sets.

Examples:

In the previous example, the set S_3 is the only preferred extension.

In Example 6, Section 4.1.2, $\{E, C, A, F\}$ is the only preferred extension.

In Example 7, Section 4.1.2, $\{C, D, E\}$ is the only preferred extension.

In Example 8, Section 4.1.2, $\{D, T_1\}$ is the only preferred extension.

Note that there exist many other semantics for acceptability in a bipolar AF (see [MCLS05]).

 $^{^{22}}$ In a bipolar AF, another notion of conflict can be defined; it is inspired by [Ver02] and by the definition of a controversial argument proposed in [Dun95]. This notion is described in [CLS04, MCLS05] and it avoids putting

together two arguments which are in conflict about a third argument (one defeats it and the other supports it).

 $^{^{23}}$ In the sense of Definition 24.

6.2.3 Status of the arguments

Using the acceptability in a bipolar argumentation system, the acceptable arguments, the rejected arguments and the arguments in abeyance can be defined with the same method as the one used in Definition 11, Section 6.1. For example, with the preferred semantic:

Definition 30 Let A be an argument. A is acceptable (resp. rejected) iff $\exists E \subseteq \mathcal{A}$ (resp. $\not\exists E \subseteq \mathcal{A}$) a preferred extension in the sense of Definition 29 such that $A \in E$.

Note that, with this definition, there is no argument in abeyance.

Examples:

In Example 6, Section 4.1.2, A is acceptable. In Example 7, Section 4.1.2, A is rejected. In Example 8, Section 4.1.2, T_1 is acceptable.

Note also that there exist many different definitions for the status of an argument²⁴, most of them have been defined in the case of a basic argumentation system, but they may be adapted in the case of a bipolar argumentation system.

6.3 Characterizing bipolarity in the selection of the acceptable arguments

In this step, bipolarity appears in two points:

- the data used for the selection may be bipolar (for example, when the argumentation system is a bipolar argumentation system as defined in Section 4),
- the result of the selection step is bipolar: it is the partition of the set of arguments in at least two subsets (set of the acceptable arguments, set of the rejected arguments²⁵). Moreover,
 - either the rejected arguments are defined from the acceptable arguments (case of inference),
 - or the acceptable arguments and the rejected arguments are defined independently (case of the system for handling desires).

In this step, the following properties occur:

- exclusivity: an argument cannot be at the same time acceptable and rejected;
- duality: in all cases, a weak duality exists due to the partition of the set of the arguments in at least 2 subsets: accepted arguments, rejected arguments; so, an accepted argument cannot be rejected and vice-versa;
- no exhaustivity: an argument may be neither acceptable, nor rejected (it will be called "in abeyance");
- the computation of acceptable and rejected arguments is made on the same data (sometimes on bipolar data), but not always with the same process.

Bipolarity in the selection of the acceptable arguments is a type 2 bipolarity with a particular property of exclusivity.

 $^{^{24}}$ Definition 30 corresponds to the credulous acceptability of an argument, but we can define also the sceptical acceptability of an argument, and many other kinds of acceptabilities.

 $^{^{25}}$ Sometimes, there exists a third set, the set of the arguments in abeyance which is defined from the two other sets.

7 Reasoning with acceptable arguments

The last step of an argumentation process consists in deciding when a conclusion is inferred. In the case of inconsistency handling in knowledge bases, for instance, one might find the most plausible inferences. In a decision context, one might find the best decisions. Such conclusions are defined on the basis of the different categories of arguments defined in the previous section, thus on the basis of bipolar data. Let us illustrate this in the case of inconsistency handling. One can imagine different criteria for inference. Let $\mathcal{K} = \{k_j; j = 1, \ldots, l\}$ be a base representing the available knowledge of an agent, and $\mathcal{A}_{\mathcal{K}}$ the set of all explanatory arguments that can be constructed from \mathcal{K} .

Definition 31 A formula h is inferred from \mathcal{K} iff:

- there exists at least one acceptable argument $\langle H, h \rangle \in \mathcal{A}_{\mathcal{K}}$, or
- all the arguments supporting h are acceptable, or
- there exists an acceptable argument $\langle H, h \rangle \in \mathcal{A}_{\mathcal{K}}$ and there is no rejected argument $\langle H', h \rangle$.

In this step, there is no bipolarity, but for the fact that we use bipolar data for reasoning. The first criterion proposed by Definition 31 corresponds to the doctrine that casuists called *probabilism* and represents a credulous reasoning. The second criterion represents a skeptical or cautious reasoning and the third one is an intermediary one.

But we could also have other intermediary criteria. Assume that we could have a complete preordering between arguments that reflects the impact²⁶ on the arguments of their defeaters and their defenders (and their supporters, in the case of the bipolar framework described in Section 4.1.1). Then we could use what casuists called *probabiliorism*: take h as inferred only if it is a conclusion of the "best" arguments in the sense of the preordering.

Note that, in a bipolar framework, we could also have *two preorderings*²⁷: one for the ordering with respect to support and defence, the other for the ordering with respect to defeat (and irrelevance objections, if our frameworks were dealing with that kind of defeat). If you are cautious, you prefer undefeated arguments or less defeated arguments, if you are looking for positive information, you prefer supported and defended arguments for your inferences. If you are looking for fundamental presuppositions, you will take the cautious method, but if you are looking for advance in research, you will choose the positive one.

8 Conclusion

This paper presents a comprehensive survey of the existence of bipolarity in argumentation frameworks. Indeed, it shows that bipolarity may appear at different levels of an argumentation process. However, this broadly depends on the considered application.

- At the object level, bipolarity does not always appear. It is the case in argumentationbased inference systems where arguments are constructed in favour of and against a given conclusion. Even if the two kinds of arguments play different roles, their logical definitions are similar. In argumentation-based decision, things look different since two kinds of arguments are built and handled differently: arguments in favour of a decision and arguments against a decision.
- At a meta level, arguments may have supporters and also defeaters. Support and defeat relations define a bipolar interaction useful to define the strengths of arguments. So, in this case, we propose an extension of Dung's framework: an abstract bipolar argumentation framework.

 $^{^{26}\}mathrm{It}$ is the case of the preorderings issued of the valuation step described Section 5.

²⁷But, it is not the case in this document.

- Then, using a bipolar argumentation framework, we can propose some bipolar valuations which take into account the two kinds of interaction.
- Finally, the result of an argumentation process may be presented in a bipolar way: acceptable arguments and rejected arguments (the arguments in abeyance being derived from the two previous classes).

We summarize the discussion presented in this paper on the following table:

Criteria	Step 1		Step 2	Step 3		Step 4	
	(arguments)		(interactions)	(values)		(acceptability)	
	Case 1	Case 2		Local	Global	Case 1	Case 2
Exclusivity	Х	Х	Х	Х	_	Х	Х
Duality	Х	Х	Х	Х		Х	Х
Exhaustivity				Х	Х		
Computation of the same data	Х	Х	X/—	Х	Х	Х	Х
Same computational process	Х					Х	
Use of scales			_	X(1)	X(2)		
Type of bipolarity	2	2	2 or 3	$3 \rightarrow 1$	3	2	2

X: criterion is verified, —: criterion is not verified,

X/—: criterion may be either verified or not verified

(1) one bipolar univariate scale

(2) two unipolar bivariate scales

At all levels, underlying or introducing bipolarity leads to enriching and improving the power of the argumentation process. It also reflects the need for bipolar information in many real applications (see many discussions about this subject [Bou94, TP94, LVW02, BDKP02]).

There are many possible future works:

- First of all, the complete study of the abstract bipolar argumentation framework (in particular, the computational aspect of acceptability in this framework).
- A second extension of this work would be to study the impact of basing an argumentation system on bipolar data (knowledge and preferences²⁸) on the kinds of bipolarity highlighted in this paper. For instance, the following questions should get answers: if arguments are built from bipolar data, do we still have the same kind of bipolarity at the level of arguments? can this have an impact on the kind of interactions that may exist between arguments? are the arguments still evaluated in the same way? finally, do we still have the two kinds of bipolarity at the level of acceptability?
- Another extension of this work would be the study of argumentation-based decision models when bipolar data are used. In [AP04c, ABP05], argumentation-based decision models have been proposed for respectively making decisions under uncertainty and for multiple criteria decision making. However, in both models, the arguments are built from a knowledge base and a base containing the goals of an agent. We will extend those models by taking into account bipolar data. This may result in richer models and possibly better decisions.

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²⁸For example, belief bases which are partitioned in three parts: a positive part, a negative part and possibly a "neutral" part.

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