Simulation of Turing machines in CL

Computing Numeric Functions with Turing machines Coding of Data

We treat the **tape** as a **stack** containing numbers in **monadic** notation:

$$1^{s_n}21^{s_{n-1}}2\cdots 1^{s_1}21^{s_0}\underline{2}$$

The numbers on the stack are $s_n, s_{n-1}, \cdots, s_1, s_0$; s_0 is the **top**.

The **call** of the n-ary function $f(x_1, \ldots, x_n)$ can be *computed* by starting the computation with the arguments **pushed** onto stack s:

$$s1^{x_1}21^{x_2}2\cdots 1^{x_{n-1}}21^{x_n}\underline{2}$$

The **result** replaces the arguments:

$$s1^{f(x_1,\ldots,x_n)}\underline{2}$$

Coding of Turing Instructions

Turing machines are **composed** from **six** instructions. Together with a Nop they can be coded by **triples** of dyadic numbers: $Nop = 7$, $R = 8$, $L = 9$, $W_1 = 10$, $W_2 = 11$, $Wh_i = 12$, $If_i = 13$
The last two need to encode additional arguments $Wh_1(p)$ and $If_1(p, q)$. For that we need **padding** $Pad(n) = 21^{n-1}$:

$$Wh_1(p) = p \star \overbrace{21^{n-1}}^{\text{pad}} \star Wh_i \qquad \text{where } |p|_d = n$$
$$If_1(p, q) = Nop^i \star p \star Nop^j \star q \star 21^{n-1} \star If_i$$

where
$\max(|p|_d, |q|_d) = n = 3{\cdot}i + |p|_d = 3{\cdot}j + |q|_d$

Instructions are **concatenated** in **reverse** order: **left a block** macro $Lb_1 \equiv L\,Wh_1(L)$ is coded as

$$Wh_1(L) \star L = \overbrace{121}^{L} \star \overbrace{211}^{\text{pad}} \star \overbrace{212}^{Wh_i} \star \overbrace{121}^{L} = 5417$$

Decoding of instructions

The function $Instr(p) = Take(3, p)$ with a single clause
$Instr(8{\cdot}q + i) = i \leftarrow 7 \leq i \wedge i \leq 14$ yields the first instruction (which is stored in reverse) of the program p.
The function $Next_i(p) = Drop(3, p)$ with a single clause
$Next_i(8{*}q{+}i) = q \leftarrow 7 \leq i{\wedge}i \leq 14$ yields the remainder of the program (if non-empty). Thus $p = Next_i(p) \star Instr(p)$
Ifs and whiles are **decoded** by

$$Next\_if_1(r \star If_1(p, q)) = r \star p$$
$$Next\_if_2(r \star If_1(p, q)) = r \star q$$
$$Next\_wh_1(r \star Wh_1(p)) = r \star Wh_1(p) \star p$$
$$Next\_wh_2(r \star Wh_1(p)) = r$$