

Modified Cantor's Pairing function

$Tr(n) = 1 + \sum_{i=0}^n i$ is called the **triangular function**. If we define:

$$\langle x, y \rangle = Tr(x + y) + x$$

then the function satisfies the pairing property:

$\langle x, y \rangle$	0	1	2	3	4	5	6	...
0	1	2	4	7	11	16	22	...
1	3	5	8	12	17	23	30	...
2	6	9	13	18	24	31	39	...
3	10	14	19	25	32	40	49	...
4	15	20	26	33	41	50	60	...
5	21	27	34	42	51	61	72	...
6	28	35	43	52	62	73	85	...
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	

We write $x; y$ instead of $\langle x, y \rangle$ and abbreviate $a; (b; c)$, i.e. $\langle a, \langle b, c \rangle \rangle$, to $a; b; c$

We then have

$$x; y = \frac{(x + y) \cdot (x + y + 1)}{2} + x + 1$$