The Method of Propositional Tableaux

Notation. We let all binary propositional connectives group to the right. We assign the highest precedence to the negation $\neg$. Next lower precedence has the conjunction $\wedge$ and then the disjunction $\vee$. The connectives of implication $\rightarrow$ and equivalence $\leftrightarrow$ have the lowest (and same) precedence. For instance, the formula $\varphi_1 \rightarrow \varphi_2 \leftrightarrow \varphi_3 \vee \neg\varphi_4 \wedge \varphi_5$ should be read as the formula $(\varphi_1 \rightarrow (\varphi_2 \leftrightarrow (\varphi_3 \vee ((\neg\varphi_4) \wedge \varphi_5))))$.

Tableau expansion rules.

$$\frac{\neg\varphi}{\varphi *} \qquad \frac{\varphi \wedge \psi}{\begin{array}{c}\varphi\\ \psi\end{array}} \qquad \frac{\varphi \vee \psi}{\varphi \quad \psi} \qquad \frac{\varphi \rightarrow \psi}{\varphi * \quad \psi} \qquad \frac{\varphi \leftrightarrow \psi}{\begin{array}{c}\varphi \rightarrow \psi\\ \psi \rightarrow \varphi\end{array}}$$

$$\frac{\neg\varphi *}{\varphi} \qquad \frac{\varphi \wedge \psi *}{\varphi * \quad \psi *} \qquad \frac{\varphi \vee \psi *}{\begin{array}{c}\varphi *\\ \psi *\end{array}} \qquad \frac{\varphi \rightarrow \psi *}{\begin{array}{c}\varphi\\ \psi *\end{array}} \qquad \frac{\varphi \leftrightarrow \psi *}{\varphi \rightarrow \psi * \quad \psi \rightarrow \varphi *}$$

Examples of tableaux.

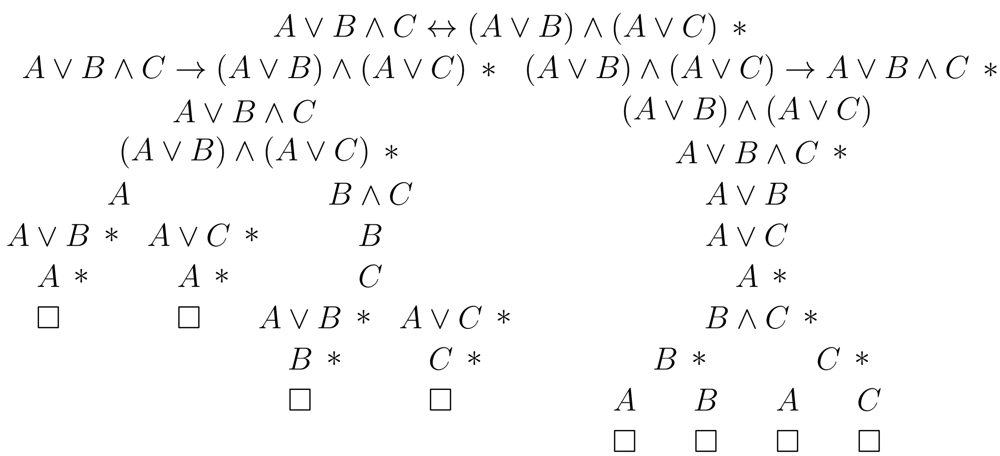

Figure 1: Distributivity of logical disjunction over logical conjunction

$$(A \to B) \wedge (B \to C) \wedge A \to C \; *$$
$$(A \to B) \wedge (B \to C) \wedge A$$
$$C \; *$$
$$A \to B$$
$$(B \to C) \wedge A$$
$$B \to C$$
$$A$$
$$\begin{array}{ccc} A \; * & & B \\ \square & B \; * & C \\ & \square & \square \end{array}$$

Figure 2: Transitivity of logical implication

$$(A \to B) \leftrightarrow \neg B \to \neg A \; *$$
$$\begin{array}{cc} (A \to B) \to \neg B \to \neg A \; * & (\neg B \to \neg A) \to A \to B \; * \\ A \to B & \neg B \to \neg A \\ \neg B \to \neg A \; * & A \to B \; * \\ \neg B & A \\ \neg A \; * & B \; * \\ A & \begin{array}{cc} \neg B \; * & \neg A \\ B & A \; * \\ \square & \square \end{array} \\ B \; * & \\ \begin{array}{cc} A \; * & B \\ \square & \square \end{array} & \end{array}$$

Figure 3: Contraposition of logical implication