The Method of Propositional Tableaux

Notation. We let all binary propositional connectives group to the right. We assign the highest precedence to the negation $\neg$. Next lower precedence has the conjunction $\wedge$ and then the disjunction $\vee$. The connectives of implication $\rightarrow$ and equivalence $\leftrightarrow$ have the lowest (and same) precedence.

Tableau expansion rules.

$$\frac{\neg\varphi}{\varphi *} \qquad \frac{\varphi \wedge \psi}{\begin{array}{c}\varphi \\ \psi\end{array}} \qquad \frac{\varphi \vee \psi}{\varphi \quad \psi} \qquad \frac{\varphi \rightarrow \psi}{\varphi * \quad \psi} \qquad \frac{\varphi \leftrightarrow \psi}{\begin{array}{c}\varphi \rightarrow \psi \\ \psi \rightarrow \varphi\end{array}}$$

$$\frac{\neg\varphi *}{\varphi} \qquad \frac{\varphi \wedge \psi *}{\varphi * \quad \psi *} \qquad \frac{\varphi \vee \psi *}{\begin{array}{c}\varphi * \\ \psi *\end{array}} \qquad \frac{\varphi \rightarrow \psi *}{\begin{array}{c}\varphi \\ \psi *\end{array}} \qquad \frac{\varphi \leftrightarrow \psi *}{\varphi \rightarrow \psi * \quad \psi \rightarrow \varphi *}$$

Examples of tableaux.

$$\begin{array}{cccccccc}
\multicolumn{8}{c}{A \vee B \wedge C \leftrightarrow (A \vee B) \wedge (A \vee C) \ *} \\
\multicolumn{4}{c}{A \vee B \wedge C \rightarrow (A \vee B) \wedge (A \vee C) \ *} & \multicolumn{4}{c}{(A \vee B) \wedge (A \vee C) \rightarrow A \vee B \wedge C \ *} \\
\multicolumn{4}{c}{A \vee B \wedge C} & \multicolumn{4}{c}{(A \vee B) \wedge (A \vee C)} \\
\multicolumn{4}{c}{(A \vee B) \wedge (A \vee C) \ *} & \multicolumn{4}{c}{A \vee B \wedge C \ *} \\
\multicolumn{2}{c}{A} & \multicolumn{2}{c}{B \wedge C} & \multicolumn{4}{c}{A \vee B} \\
A \vee B \ * & A \vee C \ * & \multicolumn{2}{c}{B} & \multicolumn{4}{c}{A \vee C} \\
A \ * & A \ * & \multicolumn{2}{c}{C} & \multicolumn{4}{c}{A \ *} \\
\square & \square & A \vee B \ * & A \vee C \ * & \multicolumn{4}{c}{B \wedge C \ *} \\
 & & B \ * & C \ * & \multicolumn{2}{c}{B \ *} & \multicolumn{2}{c}{C \ *} \\
 & & \square & \square & A & B & A & C \\
 & & & & \square & \square & \square & \square
\end{array}$$

Figure 1: Distributivity of logical disjunction over logical conjunction

$$
\begin{array}{c}
(A \to B) \land (B \to C) \land A \to C \ * \\
(A \to B) \land (B \to C) \land A \\
C \ * \\
A \to B \\
(B \to C) \land A \\
B \to C \\
A \\
\begin{array}[t]{cc}
\begin{array}[t]{c} A \ * \\ \square \end{array} &
\begin{array}[t]{c} B \\ \begin{array}[t]{cc} \begin{array}[t]{c} B \ * \\ \square \end{array} & \begin{array}[t]{c} C \\ \square \end{array} \end{array} \end{array}
\end{array}
\end{array}
$$

Figure 2: Transitivity of logical implication

$$
\begin{array}{c}
(A \to B) \leftrightarrow \neg B \to \neg A \ * \\
\begin{array}[t]{cc}
\begin{array}[t]{c}
(A \to B) \to \neg B \to \neg A \ * \\
A \to B \\
\neg B \to \neg A \ * \\
\neg B \\
\neg A \ * \\
A \\
B \ * \\
\begin{array}[t]{cc} \begin{array}[t]{c} A \ * \\ \square \end{array} & \begin{array}[t]{c} B \\ \square \end{array} \end{array}
\end{array}
&
\begin{array}[t]{c}
(\neg B \to \neg A) \to A \to B \ * \\
\neg B \to \neg A \\
A \to B \ * \\
A \\
B \ * \\
\begin{array}[t]{cc} \begin{array}[t]{c} \neg B \ * \\ B \\ \square \end{array} & \begin{array}[t]{c} \neg A \\ A \ * \\ \square \end{array} \end{array}
\end{array}
\end{array}
\end{array}
$$

Figure 3: Contraposition of logical implication