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Chapter 9

Qualitative Modeling

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9.1 Introduction

Qualitative modeling concerns representation and reasoning about continuous aspects of entities and systems in a symbolic, human-like manner. People who have never heard of differential equations successfully reason about the common sense world of quantities, motion, space, and time. They do so often in circumstances offering little information, using the ability to characterize broad categories of outcomes to ascertain what might happen. For many tasks this is enough: Knowing that a valuable fragile object might be pushed off a table is sufficient reason to rearrange things so that it cannot happen. For other tasks, knowing the possible outcomes suggests further analyses, perhaps involving more detailed models. For example, an engineer designing a tea warmer must keep the tea at a drinkable temperature, while not allowing it to boil. Reasoning directly with qualitative models can capture important behavior patterns, automatically producing descriptions that are closer to the level of what people call insights about system behavior, making them useful for science, engineering, education and decision-support. Capturing the representational and reasoning capabilities that enable robust reasoning about continuous systems is the goal of qualitative modeling.

Qualitative modeling is today most commonly referred to in the literature as qualitative reasoning, but we use qualitative modeling here to emphasize that the representational work in this area shared equal importance with work on reasoning techniques per se. (As will be seen below, the tradeoffs in them are deeply intertwined.) Qualitative physics has often been used for research in this area as well, since understanding physical systems has been a central focus of much of the work in the area. However, this term has become less popular as the applicability of these ideas to areas such as finance, ecology, and natural language semantics have been explored.

We start by outlining some of the key principles of qualitative modeling, and sketch the kinds of reasoning that is involved. This sketch provides the terminology and basis for the summaries of the key ideas that constitute the bulk of this chapter. Specifically, we summarize the key ideas in qualitative mathematics, ontologies for organizing qualitative knowledge, causality, compositional modeling, states and simulation, and qualitative spatial reasoning. We close with a few examples of applications of qualitative modeling, to illustrate how these ideas play out in real examples.

9.1.1 Key Principles

There are three key principles that govern qualitative modeling.

Discretization

Qualitative representations quantize continuous properties. Discretization provides two functions. First, it turns a continuous media into entities, things which can be represented and reasoned about symbolically. Second, it provides a means of abstraction: Instead of a continuous parameter taking on an infinity of possible values, for example, one might represent its value via its sign (i.e., is it positive, negative, or zero?), or via comparison with important other values. Abstraction is crucial because qualitative modeling needs to work in situation where few if any details are known. If a rubber ball is dropped onto a hard wood floor, we know that it will bounce, without knowing the specific coefficient of restitution for the particular rubber used in the ball nor knowing the details of the stiffness of the wood in the floor. Qualitative models are focused on inferring as much as possible from minimal information.

Relevance

The discretizations chosen for qualitative representations are imposed via constraints from both the nature of the system and the reasoning to be done about it. That is, qualitative values are constructed to be relevant for some class of tasks. In reasoning about the thermal properties of a fluid, for example, the freezing point and boiling point of that substance are natural comparisons to make, defining three ranges[1] for the value of temperature for that fluid. Similarly, the regions within which a vehicle of a given type might move represent a useful qualitative distinction for reasoning about an off-road driving situation. Within a specific region constituting a qualitative value, the behavior of the system is the same, with respect to some task-specific criteria. For example, if one is only concerned with knowing whether or not a fluid is solid, liquid, or gas, every specific numerical value of temperature between the freezing point and the boiling point are equivalent. But if one also wants the fluid to be drinkable, there are further subdivisions imposed by that task upon temperature.

Ambiguity

Working a high level of abstraction has a cost: There often is not enough information to ascertain which of several possible behaviors will occur. That is, the predictions made by qualitative models are often ambiguous. This makes qualitative models an ideal complement to traditional mathematical and numerical techniques. Traditional techniques require someone to first frame the problem, by identifying what phenomena are relevant and what categories of behaviors are conceivable. Mathematical models for the relevant phenomena can then be used to ascertain exactly what behaviors will

[1] Ignoring high pressure situations at which both the freezing point and the boiling point are the same, known as the triple point in thermodynamics.

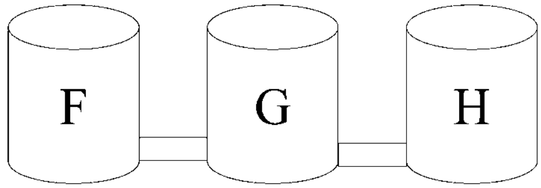

Figure 9.1: Three containers.

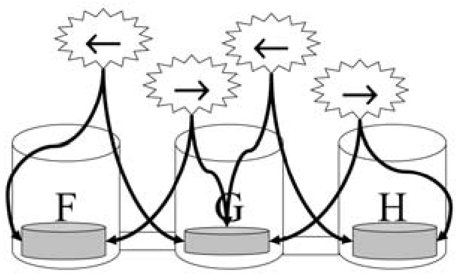

Figure 9.2: Model formulation leads to new conceptual entities, including processes.

occur, up to the resolution and accuracy of the models used. Qualitative models formalize this framing process, via automatic modeling algorithms to identify relevant phenomena and identify conceivable kinds of behaviors.

9.1.2 Overview of Basic Qualitative Reasoning

To ground our subsequent discussion, we outline how fundamental steps in qualitative reasoning fit together to construct a description of possible behaviors. We use the three containers example, shown in Fig. 9.1, as an illustration.

Model formulation

The first step in reasoning is to construct a model of the system or situation. The input description is typically called the *scenario*. The knowledge of the kinds of entities and phenomena that can occur are represented as *model fragments*, typically stored in a library called the *domain theory*. A model for the scenario is assembled from relevant model fragments via a reasoning process called *model formulation*. Model formulation uses both the contents of the scenario and constraints imposed by the task for which the model is being constructed.

In our example, the domain theory might include model fragments for describing the properties of pieces of liquid within a container, and the possibilities of flows between them, if the pipes are open. We depict instances of these entities graphically in Fig. 9.2, using pieces of water for the contained liquids and starbursts for the possible flows that might, depending on circumstances, occur.

Elaborating a qualitative state

The scenario model consists of a set of model fragments, representing properties and relationships that may or may not hold at any given time. The set of entities that are

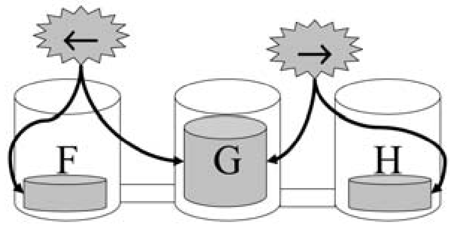

Figure 9.3: Active processes given assumed initial levels.

held to exist by the model and what parameters they have are defined by the set of active model fragments. The qualitative values of the parameters are typically only partially constrained by the set of active model fragments. A *qualitative state* is defined by the set of active model fragments plus the qualitative values for all of the parameters of the system.

If the scenario describes some specific initial condition, then once a scenario model has been constructed one or more qualitative states describing that scenario can be derived. Notice that there can be more than one qualitative state describing the scenario because the initial scenario description might be incomplete. This is often the first step in understanding a complex system, for monitoring or diagnosis. The causality imposed by qualitative models can be important, since achieving desired states and avoiding undesirable states requires tracing back through the antecedents for the state to manipulable aspects of the system. For example, it is the low level of fuel in the tank that causes a warning light in a car to come on; to extinguish the warning light requires adding fuel to the tank.

Returning to our running example, suppose the level of water is higher in G than it is in either F or H. Then, assuming the pipes are open, there will be two instances of water flows, representing water leaving G, as shown in Fig. 9.3. While this example looks simple, it involves some surprising subtleties. For example, our inference that water is flowing out of G rests on the heights of the bottoms of the containers all being the same: if H were much higher, its pressure would be higher and flow would go in the reverse direction in that path. If we modeled gasses in the containers and they were closed, then we would have to take their contribution to the pressure into account. The ability to reason about different modeling assumptions is discussed below.

Qualitative simulation

Some qualitative states can last forever, but most do not. Qualitative simulation identifies what states can happen next. This process can be applied recursively, to derive all of the states that can follow from a given initial qualitative state. Generating all possible categories of behaviors is called *envisioning*. For very simple systems envisioning can be polynomial in the complexity of the qualitative spatial model used, but in general it is exponential in the number of constituents of the qualitative state. When landmark introduction is used, the number of qualitative states can be infinite even for simple systems. This means that the choice of qualitative representations and what is needed in terms of predictions must be considered carefully when designing reasoning systems for a specific task.

Returning one last time to our three containers: Fig. 9.4 summarizes the envisionment for the situation in Fig. 9.3. Small arrows indicate liquid flows inside a state

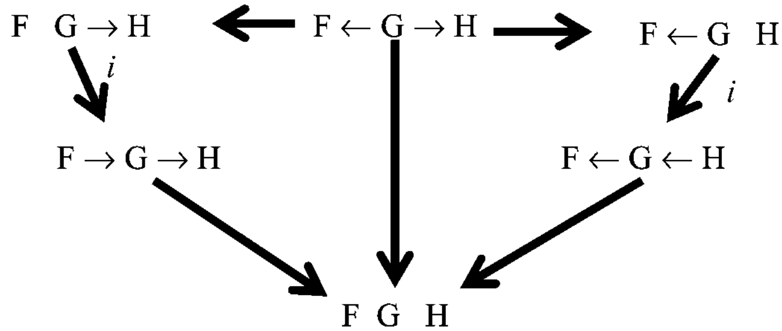

Figure 9.4: An envisionment for the three containers situation, starting from the qualitative state of Fig. 9.3.

between two containers, big arrows indicate transitions. The state in the middle represents the qualitative state depicted in Fig. 9.3.

Notice that we actually do not know which of the two flows might stop first. Although the pipes have been drawn the same size, we do not know if one of them is partially clogged, for example. So there are three possible next states. Transitions marked with an "*i*" occur in an instant, whereas all others require an interval of time to occur. Thus we can see that, if either flow stops before the other, it will then reverse, stopping only when the entire system reaches equilibrium (the bottom state). From the fact that there are transitions into this state and no transitions out, we can conclude that this state will last forever, unless something else disturbs the system.

9.2 Qualitative Mathematics

Qualitative mathematics formalizes notions of quantity and relationships at a more abstract level of detail than mathematics as traditionally used in science and engineering. While causality is intimately linked with qualitative mathematics in some systems of qualitative modeling, this is not universal and so causality is discussed later.

9.2.1 Quantities

Qualitative notions of value

There is a surprising range of qualitative representations for continuous one-dimensional parameters. Any account of qualitative value must address three issues:

1. What is the set of values used? For most traditional models, parameters take on real values (i.e., elements of $\Re$) or floating point approximations thereof. Most qualitative value systems identify a finite set of values. In some systems the set of values is described statically, while in others the set of values varies dynamically, providing variable precision.

2. How can they be reasoned with? Traditional values can be plugged into equations and used to derive new values. Most qualitative systems support some form of propagation of value information through qualitative relationships, enabling information about one part of a system to be used to infer information about other parts. Some qualitative systems support more equation-like

algebraic manipulations, although the set of allowable manipulations is more restricted due to the nature of qualitative values [110].

3. How can they be generated from other sources of information? Often scenario descriptions are automatically derived from sensor data or other noisy, limited-accuracy numerical information. Techniques used for this include simple range calculations, fuzzy logic [22, 6], and trendline analyses [30].

The status abstraction The simplest notion of qualitative value is simply describing a parameter as normal/abnormal, the *status abstraction* developed by Abbott [1] to support diagnosis of aircraft engine failures and subsequently used in photocopier modeling [5]. Reasoning with such values is via propagation through a qualitative model of the system, and the values themselves are computed via tables that describe ranges of values for particular sensors that are considered nominal.

Signs Almost as simple is the sign algebra developed by de Kleer and Brown [26], where parameters are characterized as positive (+), negative, (−), or zero (0). The sign algebra is the simplest system that enables continuity constraints to be applied: A qualitative value cannot jump directly from + to −, or from − to +, without first going through 0. When applied to derivatives, it provides a natural formal expression for the intuitive idea of a parameter either being increasing, decreasing, or remaining constant. By judicious introduction of quantities during model formulation, a surprisingly wide range of systems can be modeled with this algebra. For example, if freezing can be ignored the temperature of a fluid could be modeled by the difference of its temperature with the boiling temperature, so that − corresponded to liquid and + corresponded to gas.

Signs also make concrete the central role of ambiguity in qualitative models. Consider the equation

$$[x] + [y] = [z],$$

where $[a]$ means "the qualitative value of a". (This is an example of a *confluence* [25].) If we know that x and y are +, then we know that z must be + as well. However, if we know that $[x]$ is + and $[y]$ is −, then we can say nothing about $[z]$—whether it is +, −, or 0 depends on the relative magnitude of x and y. This ambiguity has been handled in two different ways in qualitative sign algebras. One way is to introduce a new value, often labeled ?, to explicitly represent ambiguity. This provides a compact representation of the ambiguity which can then be propagated through the rest of the system. The other way is to introduce branching, characterizing the ambiguity either by creating alternate models corresponding to combinations of different values, or carrying through the model complex labels representing the possibilities, in an ATMS-like fashion [27].

Finite symbolic value systems Early efforts to characterize numerical values in AI often focused on describing parameters in terms of a small number of terms, such as `high`, `medium`, or `low`, but without much consideration about how to reason with such systems or how to construct them from numerical parameters. Defining consistent algebras for combining such values can be tricky: `high` + `high` clearly equals `high`, but does `medium` + `low` equal `medium` or `high`? Whatever system of combination is chosen must be consistent with how the qualitative values are computed from underlying information, which can be tricky. Nevertheless, such systems have important uses. For example, Guerrin [56] observes that ecology researchers gathering data have particular discretizations of this form that they find natural, and described how to create algebras that map between different resolution finite symbolic value systems. Similarly, a number of researchers have found adapting the fuzzy logic notion of overlapping values in qualitative representations to be valuable (cf. [101, 10]).

Expressiveness is one side of a tradeoff for the choice of qualitative value representation. The other side is tractability. When constructing qualitative states using parameters whose values are represented as signs, each parameter introduces three (or four, if there is an explicit ambiguity value) potential choices. If there are N parameters and M possible qualitative values for a parameter, then there are M^N possible states that are distinguished by parameter values. There are typically additional choices involved in defining states, including status of model fragments and the truth of external statements, as discussed below. Moreover, the laws governing system behavior typically rule out the vast majority of these possible states. But the point remains true: The more expressive the qualitative value representation, the less tractable qualitative simulation tends to become.

Quantity spaces, limit points, and landmarks

One limitation of the schemes outlined so far is that they have particular fixed levels of resolution. Sometimes the set of distinctions to be drawn needs to change dynamically, during the course of reasoning. Typically this happens due to some comparison between two values becoming relevant that could not have been predicted before reasoning began. Returning to fluid temperature, one might be able to determine that for a specific task, either the boiling point, the freezing point, or both might be relevant for that task, and define ranges accordingly. However, if the fluid is in contact with multiple objects (directly or indirectly), there are possible heat flows to be considered. Heat flows are conditioned on temperature differences between the entities involved. The relevant temperatures to compare against are therefore determined also by the heat flows that the fluid can potentially participate in. Consider, for example, planning the cooking of a complex meal. Many dishes will be brought to various temperatures by a variety of means, and solids and fluids placed in different locations and combined in a variety of ways. It is hard to see how a fixed vocabulary symbolic algebra could be constructed for this situation that would be small enough to be tractable. This is why many qualitative modeling systems use dynamic resolution value representations.

The *quantity space* representation for a quantity Q defines the value of Q in terms of ordinal relationships with a set of other quantities, the *limit points* for that quantity space [43]. The set of limit points is determined by what comparisons are relevant for the current task. In some qualitative modeling systems (e.g., QSIM [74, 75], GARP [13], the set of limit points is determined by the modeler. In others (e.g., qualitative process theory [43]), limit points are derived automatically on the basis of the model fragments that have been created and reasoning about the interactions in the model. For example, zero is always a limit point in the quantity space for derivatives, since the relationship of the derivative to this value determines the important property of whether a value is increasing, decreasing, or constant (*D*s values, in QP theory).

Quantity spaces can be partially ordered, which is useful for explicitly representing partial states of knowledge. One might know when cooking, for example, that both the sautéed onions and the sauce are hotter than room temperature, but may not know their relative temperatures when combining them. A *value space* is a totally ordered quantity space. Imposing a total order can be useful for qualitative simulation algorithms, since it reduces one source of ambiguity (and hence possible branching) and allows graph-like depictions of parameter values to be created for visualization. A value space with N limit points is essentially a $2N + 1$ finite symbolic algebra, with the symbols being the specific limit points and the regions above, below, and between them.

Another source of potential comparisons are *landmark values*, or *landmarks*. A landmark is a fixed (although typically unknown) numerical value. Some qualitative modeling systems (notably QSIM) introduce landmark values dynamically. For example, when a partially elastic ball bounces, energy is lost with each collision, and the maximum height it reaches on each successive bounce decreases each time. Each such maximum height can be represented as a landmark value, and the fact that the system is losing energy can be inferred from the fact that each subsequent landmark value is smaller than the previous one. It is important to note that all landmark values are limit points, but not all limit points are landmark values. All landmark values are limit points because the newly introduced distinction is used to carve up future states: Otherwise, it would not be useful to introduce them. But limit points need not be defined in terms of specific fixed values, as the temperatures in the cooking example illustrates. One can, for example, infer that two temperatures can become equal without introducing a new entity to represent what that equilibrium temperature is.

The tradeoff with landmark introduction is, again, expressiveness versus tractability. With landmark introduction, whether a system is oscillating steadily, decaying, or growing via positive feedback can be "read off" directly by comparing subsequent landmarks, in a correct qualitative simulation. However, the number of possible states grows from finite to infinite, since between two landmarks one can always introduce another one.[2] Moreover, formulating the laws governing a system so that the landmarks produced are always correct can be problematic, as discussed below.

Interval arithmetic and tolerances

A more quantitative method of providing dynamic resolution is to put numerical constraints on values. In interval arithmetic, values are represented as closed intervals whose end points are specified numerically. In tolerances, values are described as a numerical value plus a numerical tolerance, essentially a small interval around the given value within which the real value can be found. There are well-known problems with interval arithmetic, e.g., given $Z = X/Y$, with $X = [1, 2]$ and $Y = [-1, 1]$, then $Z = \{[-\infty, -1], [1, \infty]\}$. However, progress in this area (cf. [61]) may change how practical it is.

Order of magnitude representations

Sometimes effects can be ignored because they are negligible compared to others. For example, the level of water lost through evaporation can safely be ignored when com-

[2] It might even be possible to construct the reals over an interval using landmark introduction, via a method analogous to Dedekind cuts.

puting how fast the level of water is rising during a flood in a city. Such intuitions can be formalized through *order of magnitude* representations. Two distinct strategies have been used for formalizing order of magnitude knowledge. *Absolute* order of magnitude representations partition the reals into distinct equivalence classes. For example, the effect of evaporation on the water level in New Orleans would be represented in a Q-Algebra [105] as Negative Small, while the water pouring in through the levee would be represented as Positive Large. The relationships between these values would enable a reasoner to determine that the net effect will be an increase of water, all else being equal. *Relative* order of magnitude representations use a set of relationships to impose partitions dynamically. For example, in Raiman's [93] FOG, one would state that the rate of evaporation $\ll$ rate of inflow from levee, where $\ll$ is read "is negligible compared to", which would license ignoring the effect of evaporation while flooding is occurring. As with other kinds of value-based versus relation-based representation schemes, there are circumstances where each is more natural, and translations between them exist [106].

9.2.2 Functions and Relationships

Relationships between quantities express constraints imposed by the world, and describe the dynamics of a system. Just as qualitative values can be viewed as levels of abstraction over the underlying reals, qualitative mathematical relationships can be viewed as abstractions over the relationships of traditional mathematics. As before, the art is in selecting a level of representation that is appropriate for a given task, both in terms of the information available and in terms of the reasoning required.

In traditional mathematics, there is a standard distinction between algebraic relationships and integral or differential relationships. The former suffice for static situations, the latter are required for describing systems that change over time. Every modeling system that handles continuous dynamical systems always has both types of relationships, although the particular methods for handling them vary. We discuss each in turn, after focusing on compositionality.

Importance of compositionality

A hallmark of qualitative reasoning is that it handles partial information about mathematical relationships. This provides a form of *elaboration tolerance* [82]. The main tool for compositionality is defining relational primitives that express partial information about an underlying relationship, such as the use of influences in QP theory. This is the same technique used in traditional mathematics when, for instance, one uses addition to combine effects. New terms representing additional factors to include can be added when the set of models considered to be relevant changes, or correction terms can be added when models are found to be inadequate. Qualitative representations take these practices further, providing more levels of partial information, and formalizing the reasoning involved. Notice that this requires *non-monotonic reasoning*, since adding information about a relationship can change previous conclusions drawn using it.

For example, consider again reasoning about a flood. The rate of water flowing in through a breached levee will depend on a number of factors, in complex ways. There is the level of water behind the levee, the size and shape of the holes and/or gaps, and the level of water already in the city to be considered, for instance, among others. Common sense tells us some relationships already: The higher the level of water behind the levee, the faster the rate of inflow. Similarly, water will flow faster through a larger gap than a smaller one. Both of these everyday statements are constraints on the rate of water flow, which, together with the other factors, can be used to construct a function that will allow us to reason about how changes in these parameters will affect the rate of water flowing into the city. In circumstances like these, quantities are not irrelevant—if the levees had held, evacuation would not have been necessary—but it is simply not possible to create a detailed model of the situation that would allow an accurate, detailed quantitative prediction of what will happen over time. Knowing that there could be a problem, and understanding what data should be gathered to figure out how bad it is, is an essential service that qualitative models provide, formalizing what is now done intuitively and informally.

Algebraic relationships

Monotonic functional relationships play a special role in qualitative reasoning because they are the weakest relationship that enables the propagation of signs of derivatives. For example, the *qualitative proportionality* of QP theory is defined as

$$A \propto_{Q^+} B \equiv \exists f | A = f(\ldots, B, \ldots) \wedge f \text{ is increasing monotonic in } B$$

$\propto_{Q^-}$ is the same, except that f in that case is decreasing monotonic in B. (There is also a causal interpretation which is part of the definition, described in the section on causality below.) If we know that B is increasing, then, all else being equal, we know that A must be increasing. The "all else being equal" requires a closed-world assumption over the set of possible qualitative proportionalities constraining A. Such closed-world assumptions are useful for two reasons. First, they enable us to proceed with partial information. Second, if our conclusions turn out to be wrong, closed-world assumptions can be re-examined for backtracking. The function M^+, defined in [74], is similar, except that it is presumed that its arguments are the only inputs.

There is no weaker description of the relationship between two parameters that licenses the inference that "if B goes up, then A must go up". Thus monotonic functions provide an abstraction that covers a wide range of more concrete mathematical expressions, assuming that their range of validity is appropriately scoped. Such scoping is carried out in qualitative modeling systems via *model fragments* that provide explicit conditions of applicability, as discussed below.

There are times when one needs more details in combining parameters. In keeping with the goal of compositionality, the *compositional modeling language* (CML; [9]) defines compositional operators C^+, C^-, C^*, and C/, all of which are compositional in the same way that qualitative proportionalities are, e.g., one might state a one-dimensional form of Newton's Second Law as

$$C^*(F, M) \wedge C/(F, A)$$

Integral/differential relationships

To describe changes over time requires expressing relationships involving derivatives. This can be done via an explicit relationship involving derivatives. For example, the confluence [26]

$$\partial W + \partial F - \partial D = 0$$

describes how the changes in the amount of flood water in the city (W), water flooding in (F), and water flowing out through storm drains (D) might be related in a model of flooding. The confluence derivative relationship (∂) is defined as

$$\partial Q \equiv [dQ/dt], \text{ i.e., the qualitative value of the time-derivative of } Q$$

If, for example, the water flooding in (F) increases while the outflow (D) remains constant, then the water level in the city must be increasing. QP theory uses more compositional primitives to achieve the same end, through the I^+ and I^- relationships:

$$I^+(A, B) \equiv dA/dt = \cdots + B + \cdots$$

for I^-, B is a negative term in the sum. This is similar to the definition of qualitative proportionality, but differs in two important ways. First, what is constrained is the time derivative of A, not A itself. Second, the combinator is addition, rather than being unspecified. This is important because it enables knowledge of relative rates to determine the existence of dynamic equilibria. For example,

$$I^+(W, F) \wedge I^-(W, D)$$

enables us to deduce that, if D were large enough, then the city would never flood.

Tradeoffs in qualitative mathematics systems

The relative sparseness of relationship modeling choices compared to modeling choices for quantities may seem surprising. Fundamentally, the reason is that the set of analytic functions in mathematics is huge: Almost all of the useful abstractions, except for the very weakest relationships, may have already been explored by traditional mathematics.

An important question to ask is, how complete are qualitative representations relative to ordinary differential equations? By appropriate scoping, so that (mathematically) non-monotonic functions are decomposed into monotonic segments, one can create a qualitative differential equation (QDE) for any ordinary differential equation, as discussed in [75].

9.3 Ontology

Modeling systems based on traditional mathematics tend to be informal about ontological issues. Informal decisions, based on experience with the world as well as professional expertise, are used to decide what entities should be included in a situation, what phenomena are relevant, and what simplifications are sensible. One goal of qualitative modeling is to make such tacit knowledge explicit, providing formalisms that can be used for automating (either fully or partially, depending on task) the modeling process itself. For some applications, automated modeling is not necessary, and systems of qualitative mathematical equations can be constructed to do useful work, as long as the task and situations they are used in are carefully circumscribed. However, both scientifically and as a practical matter, automated modeling is of great interest. For example, in educational applications, learners typically do not have the expertise to formulate models themselves, so careful model formulation (or selection) can be essential.

There are three ontologies commonly used in qualitative modeling: *components*, *processes*, and *fields*. We discuss each in turn.

9.3.1 Component Ontologies

The component ontology is a generalization of the idea of analog electronic circuits [26]. That is, a system is considered to be a network of components. Each type of component has a defined set of terminals that can be used to connect it to others, and the only possible interactions are through such connections. Consider, for example, the circuit shown in Fig. 9.5. When the input voltage Vin rises, it causes more current to flow between the base and emitter of the transistor. This small increase of current flow causes a much larger flow between the collector and the emitter, which produces a larger voltage swing at the output Vout (which is why transistors are used as amplifiers). Note that this explanation was created by tracing through the laws associated with components, and propagating effects through their connections. In the physical world, under some conditions other kinds of interactions matter: at high frequencies shapes and distances in physical layouts matter, and at high power thermal effects must be taken into account. But for many kinds of analyses, networks of components provide an excellent way of organizing models.

While analog electronics is the paradigmatic domain for the component ontology, component models have been used in other domains, such as VLSI and chemical engineering [14]. Sometimes mixed ontologies, combining processes with components, is required (e.g., engineering thermodynamics, see [52]). In general, component models work best when the kinds of interactions there can be between entities remain relatively fixed. Modeling motion in a three-dimensional world, for example, would be an unnatural domain to use a component ontology for, since the "network" changes frequently. Component models are also poor choices when the set of entities that exists can change frequently, e.g., agent-level modeling of an ecosystem.

Bond graphs are an important category of component ontology. Structurally, bond graphs were developed as a generalization of the idea of chemical bonds, where the "molecules" become instances of components, drawn from a small library of possible types. Bond graphs have been used in a wide variety of engineering domains, and are

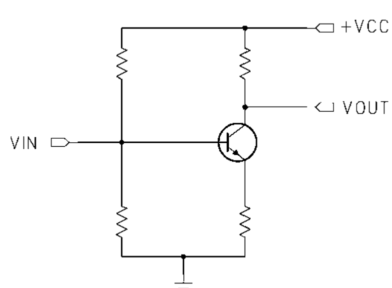

Figure 9.5: A simple electronic circuit.

attractive because of its well-worked out methodology for constructing models, most if not all aspects of which appear as applicable to qualitative modeling as to traditional modeling, although this is still being explored (cf. [83]).

9.3.2 Process Ontologies

In process ontologies (cf. [43]), processes are treated as a distinct category of entity from the other kinds of objects in the world. Processes arise from the relationships and properties of those objects, e.g., an instance of liquid flow can occur when two contained liquids are connected by an open path and the pressure of one of them is higher than the other. Note that the process is not the same as the pattern of its effects, since multiple processes can affect the same parameters. Consider a house that is losing heat to the snow outside while also being heated by its furnace inside. Whether the house is getting hotter, colder, or remains steady, as long as it is warmer than its surroundings, the heat flow out of it will continue. Thus the need to reason about multiple effects requires distinguishing a process from the outcomes it can cause.

Here is an example of a heat flow process:

```
(defmodelfragment heat-flow
  :subclass-of (physical-process)
  :participants ((the-src :type thermal-physob)
                 (the-dst :type thermal-physob)
                 (the-path :type heat-path
                   :constraints ((heat-connection
                                  the-path the-src the-dst))))
  :conditions ((heat-aligned the-path)
               (> (temperature the-src) (temperature the-dst)))
  :quantities ((heat-flow-rate :type heat-flow-rate))
  :consequences ((q= heat-flow-rate
                    (- (temperature the-src)
                       (temperature the-dst)))
                 (i- (heat the-src) heat-flow-rate)
                 (i+ (heat the-dst) heat-flow-rate)))
```

The participants represent the formal parameters of this type of process, with the type information and constraints providing sufficient conditions for deriving the existence of an instance of this type of process. Existence is not the same as acting: One can have a window that is no longer leaking heat, for example, because one has temporarily sealed it with plastic (thus making `heat-aligned` false). Reasoning about existence provides a useful intermediate stage in constructing explanations: Process instances that exist become candidates for actually doing something. An instance of a process is *active* when its conditions hold, in this case, that the temperature of the source (the-src) is higher than that of the destination (the-dst). The consequences hold only when it is active, here, that a heat flow rate, which depends on the temperature differential, acts to increase the heat of the destination and decrease the heat of the source. One can model a home heating system, for example, in terms of processes such as heat flow, liquid or gas flow, pumping, etc. (depending on the type of heating system).

Process ontologies are a natural fit to most everyday physical phenomena. Indeed, there is evidence suggesting that the notions of flow and transformations that are often encoded into natural language seem to be reasonably well described using processes [77]. However, they also have disadvantages: They require reasoning about the relationship between objects to automatically derive the existence of processes, and the dynamic nature of existence supported by the process ontology creates additional complexities in the reasoning it requires.

9.3.3 Field Ontologies

Both component and process ontologies are forms of what are called *lumped parameter* models. Many important phenomena, however, such as weather patterns and phase portraits, are spatially distributed, and cannot be understood without reasoning about that spatial structure. Field ontologies represent that structure by dividing space into regions where some parameter of interest takes on qualitatively equivalent values. This space can be physical space, e.g., for reasoning about heat transfer or meteorology, or phase space, e.g., for reasoning about dynamics, or configuration space, e.g., for reasoning about mechanical systems. For example, Yip [111] showed that qualitative reasoning about regions in phase space could lead to the automatic generation of publication-quality research results in a branch of fluid dynamics, and Bradley [12] showed that such representations could be used in designing control systems that exploit chaos to gain efficiency.

Qualitative reasoning in this ontology typically uses representations and algorithms drawn from computer vision and computational geometry to construct symbolic representations of numerical data. The most general framework, the *Spatial Aggregation Language* [3], describes the process of moving from visual representations to symbolic representations in a recursive manner. This enables lower-level symbolic constructions that still contain numerical properties (e.g., constructing iso-bar segments in weather data, see [63]) to give rise to higher-level patterns in subsequent analyses (e.g., automatically identifying pressure troughs by reasoning over the iso-bar segments).

While the state of the art in qualitative analysis using field ontologies is quite advanced, relatively little work has been done to determine the properties of qualitative simulation within this ontology. The only work to date is that of Lundell [79], who developed a spatially distributed notion of process and formulated spatial constraints for governing the process of deriving changes in regions over time. This is an area that could greatly repay further investigation.

9.4 Causality

Causality tends to be important in qualitative models because they are often formulated for tasks that involve figuring out how to change the world, such as design, monitoring, and diagnosis. The central role of causality in human explanations means that effective qualitative models for explanatory and educational purposes must be compatible with human notions of causality. The exploration of causality in complex technical domains has led to the development of more sophisticated accounts of causality in continuous systems than found in other areas of cognitive science. For example, a surprising number of models still maintain as a core constraint (inherited from classical philosophy) that a cause must always precede an effect. Empirically, people are quite happy to use causality to describe relationships that are algebraic in form (i.e., the increase in heat causes an increase in temperature, which in turn causes an increase in pressure), and do not find the simultaneity between cause and effect alarming.

There are two basic kinds of causal accounts used in qualitative modeling, *structural* and *dynamical*. We discuss each in turn.

Structural accounts of causality endow particular representational primitives with causal powers. For example, in QP theory, the *sole mechanism assumption* is that physical processes are ultimately the only source of causal changes in purely dynamical systems. These causal effects are propagated through direct influences, and then through qualitative proportionalities (which are sometimes called *indirect influences* for this reason). For example, a heat flow process directly influences the internal energy of the source and destination. If nothing else is occurring, then, since the temperatures of the source and destination are qualitatively proportional to the internal energy (aka heat, in everyday parlance) in any reasonable model, this causes the temperature of the destination to rise and the temperature of the source to fall.

In structural accounts, the relationships in qualitative mathematics are given specific causal interpretations. For example, in QP theory,

$I^+(A, B)$: B being non-zero causes A to increase, all else being equal.

$A \propto_{Q+} B$: B increasing causes A to increase, all else being equal.

This simplifies explanation generation, since describing causality within a state can be done by identifying which processes are active, describing how they cause changes to the directly influenced parameters, and then how those changes cause in turn changes in the rest of the system.

The alternative to a structural causality account is to dynamically derive casual structure. This requires choosing a place to start, identifying the beginning of the causal chain. In confluences, this is done by providing an input to the system, and viewing all changes as being caused by the effects of that input [26]. In *causal ordering*, exogenous variables are viewed as the start of causal chains, and a set of causal relationships is found by analyzing the set of (qualitative or quantitative) algebraic equations governing the system [65].

Both accounts of causal reasoning are compatible with different aspects of human causal reasoning. In many domains, causal relationships are strongly directional. Acceleration causes changes in velocity, and changes in internal energy always cause changes in temperature, and never the other way around, for example. By contrast, in an input-driven scheme, the order chosen for propagation of change can influence the direction of causality about different instances of the same component. For example, in one part of a causal explanation of the effects of a change on an analog electronic circuit, an increase in voltage across a resistor might cause the current through it to increase, whereas in another part of the same circuit, an increase in current through a resistor could cause an increase in voltage across it. Empirically, it seems most human mental models involve strongly directed causality, with analog electronics being an exceptional case. How many other domains involve reversible causality is an open question at this writing. It is important to note that strongly directed causality does not necessitate a structural account. For example, if the set of exogenous parameters governing a system being modeled through causal ordering is the same as the union of the directly influenced and uninfluenced parameters in a QP model of a system, the causal stories produced by the systems are likely to be very similar, assuming equivalent domain theories.

So far we have focused on within-state causal explanations. Across-state causal explanations describe why transitions between states occur. For example, "the increasing temperature of the water in the kettle reached its boiling point, causing it to boil." As noted above, changes in a quantity's relationships with its limit points often corresponds to a change in whether or not some model fragment is active, and hence a change in qualitative state. Thus the within-state changes that lead to the signs of derivatives involved in the comparison that changed, plus the change in the comparison itself, are viewed as the cause of the state change. (In general, there can be more than one comparison changing at once.) As always with causal reasoning, there is an implicit set of conditions that could negate it—for example, some other change might have occurred first if rates were different. Philosophically, making a distinction between these two kinds of conditions has proven difficult, but the computational grounds provided by this account provide, at least for this category of example, a clear distinction between foreground and background information that seems to match human causal explanations well.

It should be noted that this notion of causality is similar in some respects to that used by minimal-model change action frameworks (cf. [16]), in that they both provide ontological reasons for distinguishing some aspects of a situation as being more causally primitive than others, and use minimal-change heuristics (e.g., continuity in qualitative modeling) to derive potential next states. They are significantly different than probability-based accounts (cf. [91]), which are attempting to formalize conditions for inferring causal relations based on statistical information.

9.5 Compositional Modeling

Modeling is typically considered an art. One goal of qualitative modeling is to turn it into more of a science, by formalizing the process of constructing models, called *model formulation*. This involves reasoning about the entities and relationships between them in the system being modeled, the properties of the task for which the model is being constructed, and the knowledge available for modeling.

The primary methodology developed for this is *compositional modeling* [37]. The basic idea is that the knowledge available for modeling, the *domain theory*, includes a collection of *model fragments*. A model fragment is a piece of knowledge about how to model a particular entity or relationships. For example, suppose we are constructing a model of the flooding of New Orleans. One important event was a breach in the levees, which created a fluid path from the rising floodwaters to the city. The rate of flow through this path depends on a variety of factors, one of which can be considered as the fluid conductance of the path. The dependence on fluid conductance on geometry might be described as follows:

```
(defmodelfragment fluid-path-geometric-properties
 :participants ((path :type fluid-path))
 :conditions ((unblocked path))
 :consequences ((qprop (fluid-conductance path) (size path))
                (qprop- (fluid-conductance path) (length path)))
```

That is, the bigger the breach, the more fluid can potentially flow. But how should size be modeled? Perhaps that is something which can directly be ascertained from available data. But if not, it must be calculated in terms of other properties. Which properties should be used will depend on the particulars of the situation:

```
(defmodelfragment 2D-size-rectangular-estimate
  :participants ((entity :type 2D-surface))
  :conditions ((approximately-rectangular-2D-projection entity))
  :consequences ((= (size entity) (* (width entity)
                                     (height entity)))))
```

Notice that both qualitative and quantitative information can be specified in model fragments. The compositional nature of qualitative mathematics means that models appropriate for particular purposes can be assembled out of a number of such fragments, by model formulation algorithms, as described below.

One of the key problems in modeling is knowing what to include and what not to include. Quantum mechanics, for instance, is not terribly useful when considering whether or not a city might be flooded. What level of detail is relevant depends on the particular question being asked: Knowing that levees might be breached depends on estimates of how much water will build up and their state of repair, knowing when that might happen depends on estimating how quickly water is building up, and knowing where that might happen depends on knowing the detailed spatial configurations involved. Most systems can be modeled at multiple levels of detail, and from different perspectives. The information needed to make such choices is represented by explicit *modeling assumptions* and relationships among them. An important kind of relationship are *assumption classes*. An assumption class is a mutually exclusive, collectively exhaustive set of modeling alternatives for something. A model is coherent only if it includes a choice from every valid assumption class. For example,

```
(defAssumptionClass (fluid-path ?obj)
  ((consider (abstract-fluid-path ?obj))
   (consider (geometric-fluid-path ?obj))))
```

That is, for any fluid path, one should either consider its geometry or not. Choosing to consider its geometry, in turn, can lead to new assumption classes being relevant, e.g.,

```
(defAssumptionClass (geometric-fluid-path ?obj)
  ((consider (approximately-rectangular-2D-projection ?obj))
   (consider (approximately-circular-2D-projection ?obj))
   (consider (irregular-shaped-2D-projection ?obj)))
```

Notice that one of the modeling assumptions in this assumption class is the condition for the rectangular size estimation model fragment introduced above. In addition to such explicit dependencies, some compositional modeling languages define the semantics of model fragments in terms of an implicit negation, i.e., given a potential instance of a model fragment `MF`, it can only be instantiated if one can derive `(consider MF)` and/or not derive a fact of the form `(ignore MF)`.

9.5.1 Model Formulation Algorithms

Model formulation algorithms can be characterized as follows. Given

- A domain theory DT, consisting of a set of model fragments, assumption classes, and other axioms,
- A structural description SD, consisting of a set of entities and statements about them describing the structure of the system to be modeled,
- A query Q, which is a question about some aspect of the system

The output is a coherent model M such that some reasoning engine operating over M can derive a sufficiently accurate answer to Q. By coherent, we mean that the modeling choices throughout M are consistent with each other. For example, in thinking about a home heating system, one might choose to ignore properties of the system's working fluid in a question about overall thermal capacity, but then it would not make sense to include in M the relief valve used in the boiler. Any such model is called an *adequate* model. Typically there can be more than one adequate model, but in general, the more complex a model is, the more costly it is to compute with it. (Contrast, for instance, a back of the envelope calculation of a home heating system's efficiency with a computational fluid dynamics simulation of its operation over an entire winter.) Thus there is great interest in finding the *simplest* adequate model.

The original algorithm of Falkenhainer and Forbus [37] worked in two passes. First, it instantiated all of the relevant constraints by instantiating every potentially relevant model fragment from DT on SD. By using an assumption-based truth maintenance system, all sets of assumptions which would provide a model constraining the terms in Q were found. Coherence was enforced by axioms relating modeling constraints, e.g.,

```
(forAll ?sys
  (implies (and (system ?sys) (consider (black-box ?sys)))
           (forAll ?sub (implies (subsystem ?sub ?sys)
                                 (not (consider ?sub))))))
```

That is, if one is treating a system as a black box, none of its subsystems should be included in M. It was assumed that the smallest set of assumptions yielded the simplest model. The initial set of propositions were then thrown away, and only the relevant subset reinstantiated to produce M. While simple to implement, the exponential nature of the ATMS computations made it quite inefficient for large systems.

The most efficient model formulation algorithm was developed by Nayak [87], which operates in polynomial time. This algorithm is based on three assumptions:

1. Choices made in one assumption class cannot depend on choices made in others.
2. Choices in an assumption class can be partially ordered with regard to simplicity.
3. The optimality condition can be weakened from finding the simplest model to finding a simplest model.

Search for a model proceeds by walking up each assumption class implies by SD, starting with a simplest choice from each, and moving upwards until an adequate model is reached. Since the choices are independent, there is no need for backtracking due to found inconsistencies. The weaker optimality constraint means that the set of simplest satisfactory models is a surface partitioning the adequate from inadequate models, and any point on this surface is satisfactory by assumption, hence eliminating the need to optimize simplicity.

An important property of complex systems is that they typically incorporate phenomena that operate at multiple time-scales. For most of the lifetime of a building, for example, most of the interesting changes that happen to a building are best described in terms of months, years, and decades, rather than microseconds or millennia. For a particular query Q, phenomena that operate at faster time-scales can be replaced by functional relationships and phenomena that operate at slower time-scales can essentially be ignored. Rickel and Porter [95, 96] demonstrate how to use this insight in model formulation. Since the form of Q they focus on is explaining changes in a parameter (an important task for intelligent tutoring systems and explanation more generally), their adequacy criterion consists of finding at least one directly influenced parameter in the causal account constructed. They use an elegant backchaining algorithm that incrementally instantiates possible influence graphs based on the model fragments of DT, starting with the fastest time-scale, and moving to slower time-scales when an adequate model cannot be found.

9.6 Qualitative States and Qualitative Simulation

A traditional way to think about the behaviors of a complex system D consisting of a set of N continuous parameters is to define the *state space* $S(D)$ as a subset of $\Re^N$. We can define qualitative states as partitions on $S(D)$, carving it up into regions in which some set of relevant distinctions remains constant. The set of relevant distinctions includes what model fragment instances are active and the qualitative values of D's parameters. The status of model fragment instances is necessary for distinguishing qualitative states because they determine the causal constraints (including in some models quantitative equations) that govern the system. The qualitative values of parameters are important because they help determine what state transitions may occur. The difference between flood waters rising and falling, for example, is quite significant.

Since there can be multiple adequate models M of D, there can of course be multiple qualitative representations of $S(D)$. Let $QS(M)$ be the set of qualitative states implied by a model. $QS(M)$ will be finite under two conditions: (1) The set of model fragment instances must be finite and (2) the set of qualitative values for all parameters must be finite. The first condition is satisfied when the structural description of D is finite and the model fragments in DT can only create finite numbers of new individuals for any finite structural description. The second condition is satisfied if landmark introduction is not used—as noted above, landmark introduction can introduce an infinite number of qualitative distinctions.

Finite does not necessarily imply small, of course. The earliest qualitative models, which focused on modeling various kinds of motion (i.e., [24, 42]) used a small vocabulary of types of actions and qualitative decompositions of state to describe space, leading to $QS(M)$s that were polynomial in the spatial complexity of D. Suppose one has an N parameter model and uses the sign representation for qualitative values, and there are M model fragment instances, each of which can be either active or inactive. In the worst case, $|QS(M)| = 3^N * 2^M$. For large-scale engineered systems, N can be in the thousands and M can be in the hundreds. However, this worst-case estimate assumes that every parameter and model fragment are independent, whereas in reasonable domain theories, there is a strong network of constraints among them. As described below, there are applications where it is worthwhile to generate $QS(M)$ entirely, but more often, subsets of $QS(M)$ are generated incrementally on an as-needed basis.

Qualitative simulation is generating a set of qualitative states from some given initial state, constituting predictions about possible future behaviors of the system. A qualitative state can have transitions to more than one possible next state, due to the abstractness of qualitative representations. Generating all behaviors of some class is called *envisioning*. The set of all states that are possible from some initial state S is the *attainable envisionment* of S, which is a subset of the *total envisionment* of a model (i.e., $QS(M)$ itself). Typically tightly bounded subsets of $QS(M)$ are generated, but some applications (cf. [92]) require total envisionments.

An essential step in any qualitative simulation algorithm is finding transitions between states. Transitions between qualitative states occur when some condition of a model fragment changes or when a qualitative value changes. Changes in the condition of a model fragment typically reduce to changes in qualitative values (e.g., pressure equilibrates, ending a flow), and otherwise is due to an action taken to change a proposition in the model, which we will ignore for now, and focus only on value changes.

Suppose a quantity Q has limit point L in its quantity space, and in a qualitative state S, $Q < L$. For Q to reach L, it must be the case that $D(Q) > D(L)$. Transition-finding requires finding such hypothetical changes (called *limit hypotheses* in QP theory) and determining what, if any, transitions follow from them. Not all limit hypotheses lead to state transitions, because, in the absence of discontinuous changes, transitions between states must respect continuity. That is, if in state S1 $Q < L$, then there cannot be a transition directly to a state S2 where $Q > L$, since there must be some time during which $Q = L$ before. (There are ways of modeling discontinuous changes, cf. [71, 83, 84].) Transition-finding can be viewed as a constraint satisfaction problem, finding the minimal-change model from the current qualitative state in which the changes represented by a specific limit hypothesis hold, where continuity constraints are not violated, and aspects of the situation that are not causally connected to the changes are held constant. See [26, 44] and [75] for examples of algorithms.

Good qualitative simulation algorithms are complete, in that they generate the entire space of possible behaviors, but unsound, because they can include predicted futures which are not actually possible. (Kuipers [75] prefers a less intuitive formulation of these terms for qualitative simulation which enables it to be considered as sound but incomplete.) Consider a spring-block oscillator, subject to static and dynamic friction. Without friction, the envisionment of such an oscillator consists of eight states. Considering dynamic friction adds an additional state, corresponding to the block coming to rest where the spring is relaxed. Considering static friction adds two additional states, one where the block is stopped and the spring is slightly compressed, the other where the block is stopped and the spring is slightly stretched. Suppose one allows landmark introduction in reasoning about this system. Each maximal excursion of the block from the resting position of the spring then becomes a new landmark. In the physically correct qualitative simulation of this system, each subsequent landmark is closer to the resting position than the previous one. However, in the simplest spring-block oscillator formulation, there is nothing to prevent subsequent landmarks from being larger, smaller, or the same. In other words, there are paths through the set of qualitative states that do not correspond to any behavior of a real physical spring-block system, even though locally every state transition is correct.

With enough additional constraints, typically in the form of energy constraints (cf. [75]), the possible behaviors can be trimmed appropriately, for at least some systems. However, it is an open question as to how much information it takes to ensure soundness of predictions from qualitative simulation in the general case. If we take detailed numerical simulations as stand-ins for physical behavior, then there is a clear lower bound on abstractness—floating point numbers. But whether a more abstract level of representation exists that is always sufficient remains unknown. Given the ways that qualitative simulations are used, this question has proven less than urgent. Qualitative simulations are typically used to frame analyses by proposing behaviors, which are then examined as needed by more detailed models or confirmed/ruled out by data. Some spurious behaviors are, empirically, a small price to pay for the value these models provide.

9.7 Qualitative Spatial Reasoning

The ability of qualitative representations to provide a bridge between the perceptual and conceptual, by imposing discrete, symbolic frameworks on the continuous world, is perhaps most strongly evident in qualitative spatial reasoning. We start with purely qualitative representations, and then describe diagrammatic representations. The interested reader should also see the Spatial Reasoning chapter in this Handbook.

9.7.1 Topological Representations

The most fundamental qualitative representations of space are centered around topology, that is, how things are connected. Connectivity is important because it is a factor in determining whether, and how, a set of entities might interact. The best-known representation is *RCC8*, the Region Connection Calculus with 8 relationships [19]. RCC8 defines eight mutually exclusive and jointly exhaustive relationships between 2D regions: equal (=), non-tangential proper part (NTTP), tangential proper part (TTP), partially overlapping (PO), edge coupled (EC), disjoint (DC), plus the inverses NTTPi and TTPi. Intuitively, NTTP means that one thing is completely inside the other, while TTP means that the inside thing shares a surface with the outside thing, but otherwise is completely inside it. The sequence of relationships NTTP, TTP, PO, EC, DC captures the changes in connectivity as something moves from inside something to outside it, whereas the reverse sequence captures what happens when something is absorbed or ingested. A transitivity table defines what can be inferred about the relationship between regions R1 and R3, given a third region R2 and knowledge about the relationships between R1 and R2 and R2 and R3.

A variety of more complex schemes have been developed, to handle different degrees and/or dimensions of overlap, multiple piece regions, holes, and other topological phenomena (cf. [32, 17]). See [20] for an excellent survey.

9.7.2 Shape, Location, and Orientation Representations

For entities with spatial extent, their shape is one of their most fundamental properties. Qualitative shape representations focus on carving up complex objects into parts, for purposes of recognition or for ascertaining functional properties, e.g., could something serve as a handle? Hoffman and Richards [62] suggest that the human visual system uses the sign of boundary curvature as one partitioning constraint. Museros and Escrig [85] show how additional information, including relative lengths of sides and qualitative descriptions of angles, can be used with curvature decomposition to match tiles in mosaics. Nielsen [89] showed that, for reasoning about motion, shape decompositions also need to take into account mechanical constraints, such as centers of rotation.

Purely qualitative notions of orientation have been developed for a variety of purposes. For example, Kim [70] shows how representing angles in terms of quadrants and relative inclination to define a qualitative vector algebra powerful enough to reason about the motion of four-bar linkages. One of the most important uses of orientation is in creating purely qualitative descriptions of location. Freksa [54] uses orientation to introduce conceptual neighborhoods for defining locations. Clementini et al. [18] use Hernandez' [60] representation of orientation to define qualitative representations of position and distance. An alternate approach is that of Bittner and Smith [8], which defines location relative to a set of regions that partitions space (e.g., the provinces of a country), thus reducing position to qualitative topology.

9.7.3 Diagrammatic Reasoning

Qualitative spatial reasoning suffices for some tasks, but not for all. Metric information is simply necessary for some tasks: Predicting whether or not a pair of gears will bind, for instance, requires high-precision shape representations. Between these two extremes, it is often more efficient or more convenient to use metric information. Such approaches are often called *diagrammatic reasoning*, since they rely on representations that serve functional roles similar to that of diagrams or sketches in human spatial reasoning.

Metric diagram/place vocabulary model

The *metric diagram/place vocabulary* model [45] characterizes the relationship between diagrammatic and qualitative reasoning as follows. Conceptually, a metric diagram provides the same services for a reasoning system as vision does for humans: It identifies what entities are available, and provides a number of spatial operations on them that can be treated as predicates by the reasoner, although they typically are implemented by schemes that rely on, for example, computational geometry. This general-purpose input description is used to compute qualitative representations (place vocabularies) for specific tasks. In reasoning about motion through space, for example, the place vocabulary consists of regions of free space, including areas like wells where something can be trapped, depending on how much energy it has. In reasoning about kinematic mechanisms (e.g., [38, 50, 68]), the place vocabulary consists of regions of *configuration space*, i.e., the joint angles of the parts of the mechanism. For reasoning about trafficability [29], a GIS serves as the metric diagram, with the place vocabulary being the no-go/slow-go/go regions a vehicle can travel in that a terrain analyst would compute.

The Spatial Aggregation Language, described above, exploits the important insight that for many problems, there is a hierarchical set of place vocabularies, each of which rests on lower-level vocabularies.

Sketch understanding and cognitive vision

Qualitative representations provide a robust intermediate representation for handling messy perceptual inputs. Given the naturalness of hand-drawn sketches, a growing number of researchers have started to apply qualitative techniques to sketch understanding. Egenhofer [33] uses qualitative spatial topology to help formulate GIS queries from hand-drawn sketches. Hammond and Davis [57] use a qualitative vocabulary of relationships to describe representations for sketch recognition. Qualitative spatial representations have been used to reason about sketch maps [51] and for solving everyday physical reasoning problems by using analogies over sketches to formulate qualitative models [73]. Second order analogies over qualitative representations computed from sketches suffice to perform the original Evans [35] analogy task [104].

Research in cognitive vision uses qualitative representations to interpret visual data. Understanding moving objects, such as traffic patterns, is aided by imposing spatio-temporal continuity constraints via qualitative topology [21, 41]. A particularly impressive example is the learning of a table-top game from audio-visual inputs [88].

9.8 Qualitative Modeling Applications

Much of the research in qualitative modeling has been driven by applications such as those below. These are only a sample of the available papers, see the Qualitative Reasoning Workshop proceedings (available on-line at several mirror sites) and journals/conference proceedings in the relevant application areas for more details.

For simplicity, we divide up the applications and application-oriented research into three areas: Automating or assisting professional reasoning, education, and cognitive modeling. We discuss each in turn.

9.8.1 Automating or Assisting Professional Reasoning

Professional reasoning typically involves combining qualitative models with either more detailed quantitative models (e.g., early stages of design and analysis) or numerical data (e.g., late stages of design and analysis, monitoring).

Engineering problem solving

The earliest known fielded QR applications were in process control [78] and in designing photocopiers [102]. While the majority of the application efforts using qualitative modeling involve engineering domains, these are surveyed in the Model-Based Reasoning chapter of this Handbook. Consequently, we focus on areas that are more distant from the model-based reasoning community here.

Creating systems that can understanding and design mechanical systems has been one of the successes of qualitative modeling. Systems have been built that can understand mechanisms such as clocks from scanned descriptions of the parts [50], simulate mechanical designs with behavior discontinuities [98], generate designs from sketches [103], and generate innovative designs via case-based adaptation [39]. These systems use place vocabularies consisting of regions in configuration space, constructed from quantitative representations (CAD data structures, sketches) which serve as metric diagrams.

The Spatial Aggregation Language described above has been used in a variety of applications, including synthesizing thermal control strategies by deriving placements for heat sources [2], data mining in spatial data sets [94], and interpreting spatial data [64].

Economics and decision support

Informal qualitative reasoning has had a long history in economics [40], making formalized qualitative modeling a natural fit. For example, the utility of using qualitative representations to structure quantitative data is illustrated by [97], who describe how to use an order of magnitude representation to improve supervised learning for credit risk prediction. The ability to explicitly characterize categories of outcomes makes qualitative representations potentially valuable for supporting decision-making. For example, [31] illustrates how ecosystem management strategies and their outcomes can be modeled. Improving social science theories more generally, by providing formal tools for working through the consequences of theories, is another promising application. For example, [69] illustrate how to use qualitative modeling to work out consequences of a particular theory of organizational ecology.

Ecology and bioinformatics

In ecology, data can be difficult to obtain, fragmentary, and/or non-existent, making quantitative modeling often a highly speculative proposition. By capturing whole classes of behaviors, qualitative models can be produced without making as many ancillary assumptions. This has lead to an increasing interest in qualitative modeling of ecology (cf. [59, 100]), including papers by ecologists (cf. [90, 108]).

By contrast, the problem in bioinformatics is that of too much data. Here, the ability of qualitative models to characterize abstract hypotheses provides a search space of models that is more tractable for system identification from data. For example, [72] describes how to learn models of glycolysis by inductive logic programming over qualitative models. *In silico* experiments often use a variety of simulation paradigms; Trelease and Park [107] show how qualitative models of immune functions can be used to set up agent-based cellular automata simulations. de Jong and his collaborators have developed the Genetic Network Analyzer (GNA) which has been used to study genetic regulatory networks in a variety of organisms [23].

9.8.2 Education

Education is a natural application for qualitative modeling, since the closeness of qualitative models to human mental models can simplify the production of understandable explanations. In early science education, for example, most curriculum content is qualitative: What parameters and phenomena are relevant in different types of situations, and causal models interrelating them. In later science education, such concerns remain relevant, but with the additional complexity of incorporating quantitative mathematics. Formalisms for qualitative models have thus seen widespread adoption at many levels of education.

Modeling environments for education

Modeling environments can be divided into two types: Those where the primary type of modeling is qualitative, and those where qualitative models are used as one component in the modeling system. We start with the purely qualitative systems.

Concept maps are often used in education, but in a very free-form, open way. This has the advantage that it is easy to get students as young as fourth grade to generate them, but the disadvantage that it is not clear what they mean, even to the students themselves. Qualitative modeling formalisms provide a crisp but natural semantics that can be used in concept mapping tools that are both usable by students and whose models can be reasoned with, to provide coaching. For example, in the Teachable Agents project [7], a system was developed to help middle-school students create models of stream ecosystems. Students would debug their explanations by thinking of themselves as building "Betty's Brain", and would quiz the system, repairing their models until "Betty" gave the right answers. The VModel system [46] is a general-purpose concept mapping system that uses QP theory. It was designed for middle-school students to learn science by model-building, and has been used by teachers in the Chicago Public School system.

Both Betty's Brain and VModel focus on single-state reasoning, since that is sufficient for most middle-school science instruction. (For many middle-school students, mastering the idea of parameter as used in science proves quite difficult.) However, multi-state qualitative simulation is crucial for understanding more advanced phenomena. VisiGarp [11] provides an environment for students to explore multi-state qualitative simulations, filtering them according to imposed constraints and asking questions about them. Homer [80] provides tools for students to create models, which can then be explored via VisiGarp. Both of these systems, and their descendants, are being used in projects for public education, to build an understanding of sustainable development and inform policy makers as to possible consequences of different resource management decisions [99]. The challenge with such tools is that student mistakes can often lead to massive simulations, and digging through the results to figure out what went wrong can be difficult. Making the modeling environments smarter still, to characterize where the critical ambiguities are and make suggestions about what to do about them, is an important research question.

The majority of modeling environments that use qualitative models use them in combination with some variety of quantitative simulation or analysis tools. For example, Model-It [66] provides an environment for students to do systems dynamics modeling, using qualitative mathematics in the interface to provide a friendly front-end to quantitative models that are then used with a traditional numerical simulator. The Qualitative Analysis and Qualitative Simulation Laboratory [86] uses a combination of qualitative reasoning and numerical constraint reasoning to help students learn inorganic chemistry. LSDM [81] uses qualitative reasoning combined with simple numerical models to help buyers learn about different financing options. CyclePad [52] uses qualitative representations with numerical analysis and evidential reasoning to help detect inconsistent designs and recognize intended teleology in students' designs of thermodynamic cycles [36]. These systems illustrate different ways that qualitative modeling can be used to organize analyses and/or provide understandable results from quantitative data.

Self-explanatory simulators and virtual reality

Simulations can be powerful tools for education, but interpreting their results is often hard for students. Self-explanatory simulators integrate qualitative models with numerical simulations to provide causal explanations of the simulated behavior. Self-explanatory simulators for specific systems can be constructed automatically from domain theories that incorporate both qualitative and quantitative model fragments. For example, the SIMGEN compiler [47] produces a simulator runtime that looks and operates much like a traditional numerical simulator, but incorporates a compact encoding of a qualitative model that the compiler generated during the process of writing the numerical code. The simulator produces qualitative histories in addition to numerical values, for a small additional runtime cost of transition-finding (so that qualitative transitions are detected, ensuring the coherence of the qualitative and quantitative explanations) and storage for the history. Self-explanatory simulators have been used in several curricula in the Chicago Public Schools.

Virtual reality systems, either using CAVE-style immersive environments or desktop game-technology environments, have traditionally been hard to author. By mapping model fragments onto an object-oriented runtime and doing real-time reasoning about paths of interaction, qualitative models can be "assembled" as a side-effect of actions taken in a VR environment [34]. Such techniques can be used to create training systems (cf. [15]) and environments for virtual prototyping and artists (cf. [58]).

Conceptual tutoring

The potential for using qualitative modeling for coaching has only begun to be tapped. For example, [28] showed that the dependency structures created during qualitative reasoning could be manipulated to form a structure that could diagnose student errors via standard model-based reasoning, where the "components" being debugged were the ability to do particular operations or remember certain facts. In the Why2-Atlas tutoring system [67], qualitative models are being used with natural language processing to attempt to understand student explanations well enough to identify misconceptions and provide corrective feedback. These are exciting first steps at what could lead to a revolutionary technology for education.

9.8.3 Cognitive Modeling

One of the inspirations for qualitative modeling was observations of human reasoning, both about the everyday physical world and in the professional contexts of science and engineering. Unfortunately, relatively little effort has gone into using qualitative modeling to better understand human cognition, compared to more application-driven research. This is a frontier that could lead to important results on how minds work.

Mental models reasoning

Qualitative modeling has been used by a number of cognitive science efforts exploring *mental models* [55], the representations that people use in their everyday lives to understand the world around them. Kuipers and Kassirer [76] argued that some medical reasoning appears to be governed by qualitative models, based on an analysis of protocol data. Forbus and Gentner [48] used protocol evidence to argue that people use multiple models of causation in everyday reasoning. White and Frederickson [109] argue that a sequence of causal models is needed to help learners master a domain.

While existing studies of human reasoning suggest that the representations developed by the QR community may be psychologically plausible, there are reasons to doubt the psychological plausibility of purely first-principles qualitative simulation algorithms [49].

Natural language semantics

If qualitative models are part of the representational catalog used in human cognition, then one would expect to see evidence of it in many relevant aspects of human cognition. In particular, the semantics of natural language seems to be a natural place to look for such connections, given that there are similarities in the event structure representations commonly used in natural language semantics and in qualitative reasoning. It is possible to map qualitative process theory onto FrameNet [4] style conventions, and create natural language understanding systems that can construct formal qualitative representations from controlled language text [77]. This is one of the areas where a multidisciplinary approach will be needed to gain the deepest insights.

9.9 Frontiers and Resources

Qualitative modeling at this writing has a stable core of techniques, and a rapidly expanding set of applications. These applications in turn will no doubt lead to the expansion of the library of techniques over time, as new problems are discovered and addressed. While traditional application areas, such as engineering and education, remain active, part of the excitement is due to the growth of new application areas (e.g., robotics (cf. [53]), biology, cognitive modeling). The interested reader should also examine the chapter on Physical Reasoning in this Handbook, which examines logical formalizations of common sense problems.

There are a number of resources available for learning more about qualitative modeling. Most of the literature is available on-line; for example, the proceedings of the International Qualitative Reasoning workshops, which started in 1987, are available for free on-line at several mirror sites. Reference implementations of systems, including Kuipers' QSIM, Bredeweg's GARP3, Northwestern's VModel, CyclePad, and self-explanatory simulators, are available as free downloads.

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